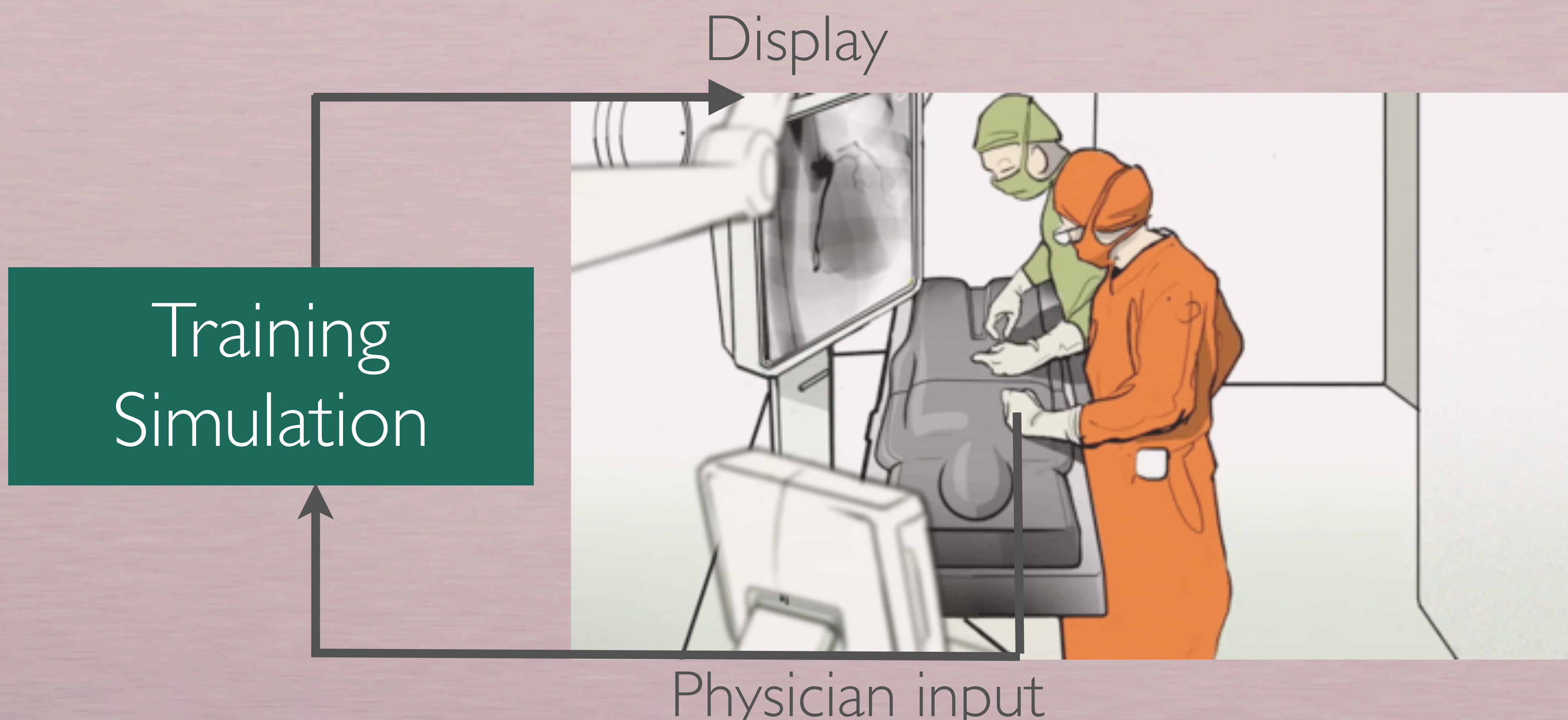
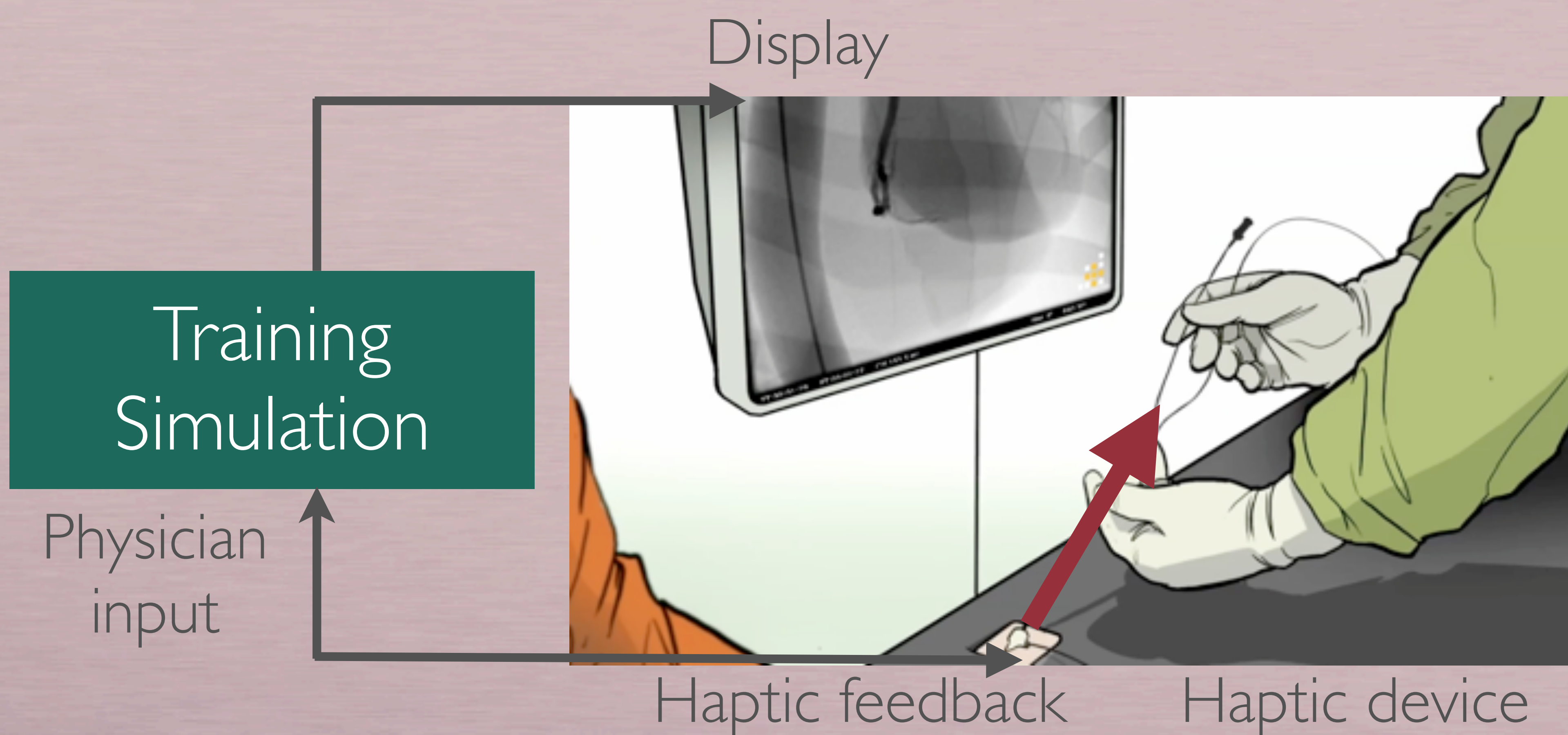
MODÉLISATION ET RÉSOLUTION DE LA RÉPONSE À LA COLLISION ET DES INTERACTIONS BIOMÉCANIQUES

Christian Duriez
INRIA Shacra-Team Lille

TRAINING SIMULATION



TRAINING SIMULATION



WHY IMPORTANT ?

- Tool-tissue interaction realism
  - Post interaction deformation
  - Behavior of the instruments
  - Haptic rendering
- Boundary conditions
  - For both tools and soft tissues
  - Interactions between anatomical structures
- Surgical simulator ≠ scripted game
  - Limited possible precomputation
  - Should allow for mistakes...
- NEED OF PHYSICS !

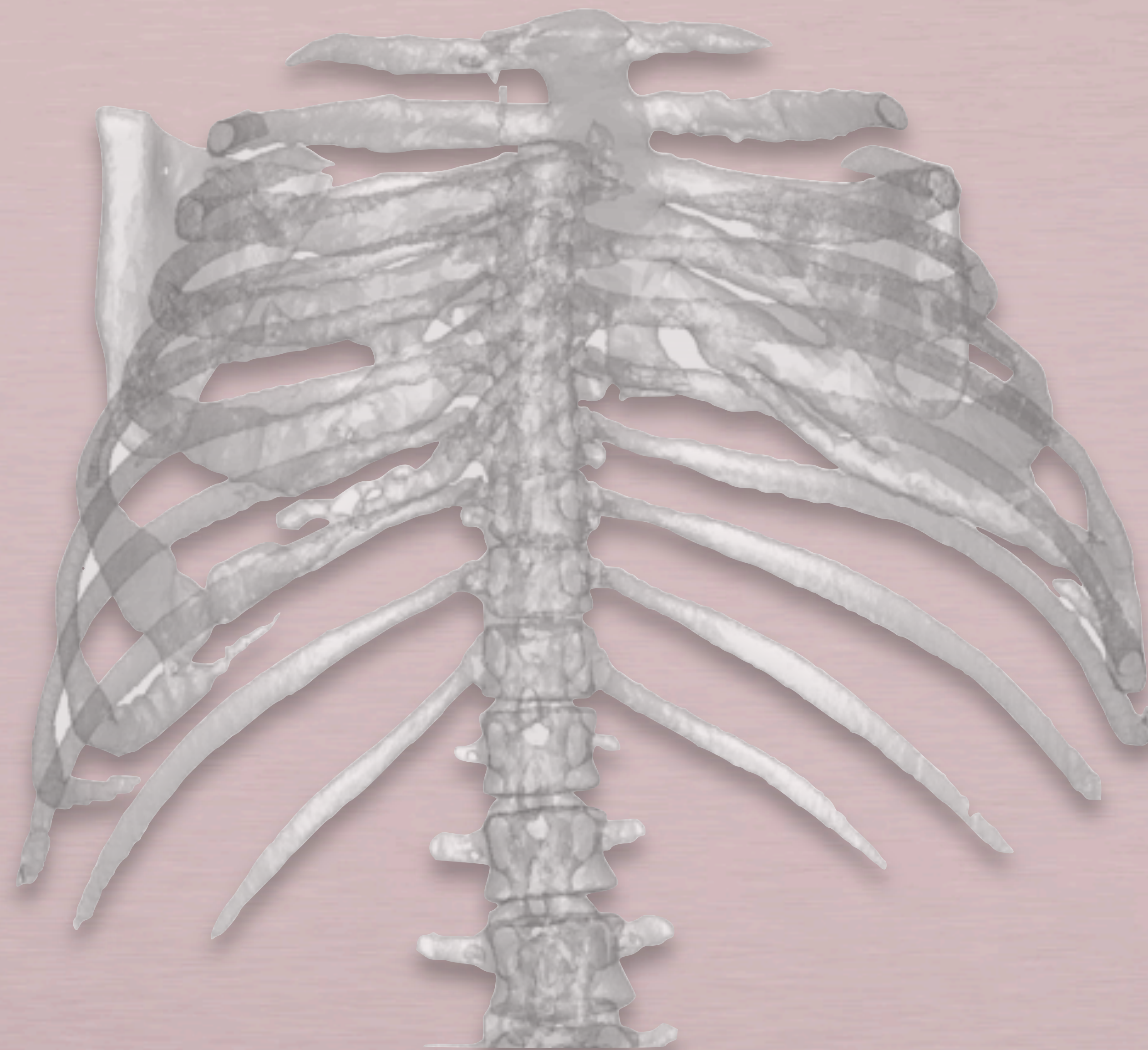
OUTCOME

- Mechanical models for real-time computation
- Constraint-based modeling of biomechanical interactions
- Haptic rendering and multithreading approaches
- Applications, ongoing research projects
- Perspective and Conclusion

- Mechanical models for real-time computation
- Constraint-based modeling of biomechanical interactions
- Haptic rendering and multithreading approaches
- Applications, ongoing research projects
- Perspective and Conclusion

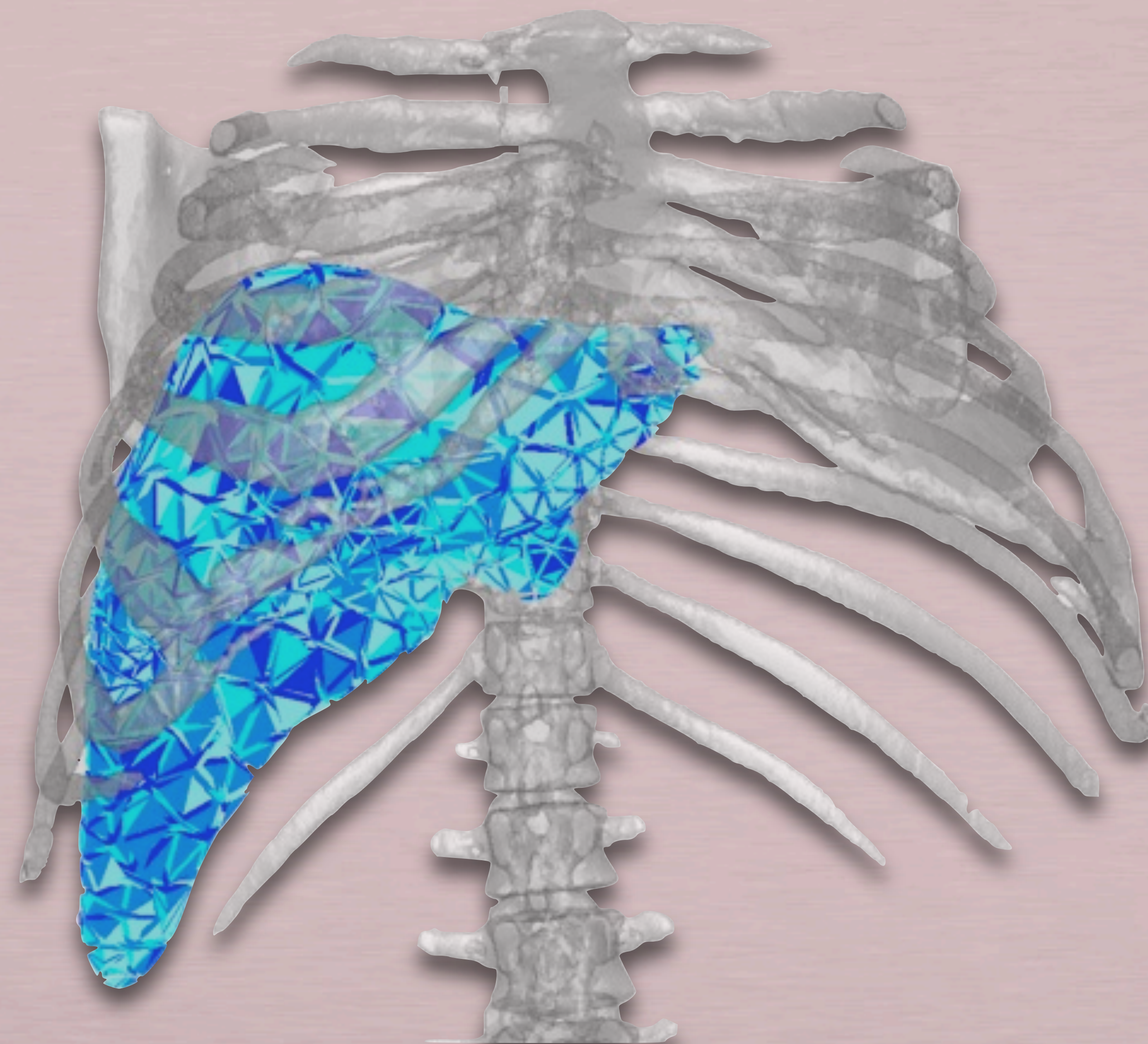
OUTCOME

- Mechanical models for real-time computation
- Constraint-based modeling of biomechanical interactions
- Haptic rendering and multithreading approaches
- Applications, ongoing research projects
- Perspective and Conclusion



MECHANICAL MODELS FOR REAL-TIME COMPUTATION

- Mechanical models for real-time computation
- Constraint-based modeling of biomechanical interactions
- Haptic rendering and multithreading approaches
- Applications, ongoing research projects
- Perspective and Conclusion



MECHANICAL MODELS FOR REAL-TIME COMPUTATION

MECHANICAL MODELS

- Newton's second law

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\,(\mathbf{q},\mathbf{v}) + \mathbf{H}^T\lambda$$

$\mathbf{q} \in \mathbb{R}^n$	Vector of generalized degrees of freedom (nodes of a deformable model)
$\mathbf{v} \in \mathbb{R}^n$	Vector of velocities
$\mathbb{M}(\mathbf{q}) : \mathbb{R}^n \mapsto \mathcal{M}^{n \times n}$	Inertia Matrix
$\mathbb{F}\,(\mathbf{q},\mathbf{v})$	Internal forces (non-linear model)
$\mathbb{P}(t)$	External forces
$\mathbf{H}^T\lambda \in \mathbb{R}^n$	Constraint force contribution

MECHANICAL MODELS

- Newton's second law

$$\mathbb{M}(\mathbf{q})\boxed{\dot{\mathbf{v}}} = \mathbb{P}(\boxed{t}) - \mathbb{F}\left(\mathbf{q}, \boxed{\mathbf{v}}\right) + \mathbf{H}^T\lambda$$

$\mathbf{q} \in \mathbb{R}^n$	Vector of generalized degrees of freedom (nodes of a deformable model)
$\mathbf{v} \in \mathbb{R}^n$	Vector of velocities
$\mathbb{M}(\mathbf{q}) : \mathbb{R}^n \mapsto \mathcal{M}^{n \times n}$	Inertia Matrix
$\mathbb{F}\left(\mathbf{q}, \mathbf{v}\right)$	Internal forces (non-linear model)
$\mathbb{P}(t)$	External forces
$\mathbf{H}^T\lambda \in \mathbb{R}^n$	Constraint force contribution

MECHANICAL MODELS

- Newton's second law

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\,(\mathbf{q},\mathbf{v}) + \mathbf{H}^T\lambda$$

$\mathbf{q} \in \mathbb{R}^n$	Vector of generalized degrees of freedom (nodes of a deformable model)
$\mathbf{v} \in \mathbb{R}^n$	Vector of velocities
$\mathbb{M}(\mathbf{q}) : \mathbb{R}^n \mapsto \mathcal{M}^{n\times n}$	Inertia Matrix
$\mathbb{F}\,(\mathbf{q},\mathbf{v})$	Internal forces (non-linear model)
$\mathbb{P}(t)$	External forces
$\mathbf{H}^T\lambda \in \mathbb{R}^n$	Constraint force contribution

MECHANICAL MODELS

- Newton's second law

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\,(\mathbf{q},\mathbf{v}) + \mathbf{H}^T\lambda$$

$\mathbf{q} \in \mathbb{R}^n$	Vector of generalized degrees of freedom (nodes of a deformable model)
$\mathbf{v} \in \mathbb{R}^n$	Vector of velocities
$\mathbb{M}(\mathbf{q}) : \mathbb{R}^n \mapsto \mathcal{M}^{n\times n}$	Inertia Matrix
$\mathbb{F}\,(\mathbf{q},\mathbf{v})$	Internal forces (non-linear model)
$\mathbb{P}(t)$	External forces
$\mathbf{H}^T\lambda \in \mathbb{R}^n$	Constraint force contribution

DICRETIZATION

- Why do we need to discretize ?
- Definition of the Degrees Of Freedom (DOF)
- Several numerical methods
- Finite Element
- Finite Difference
- ...

MODELING OF INSTRUMENTS

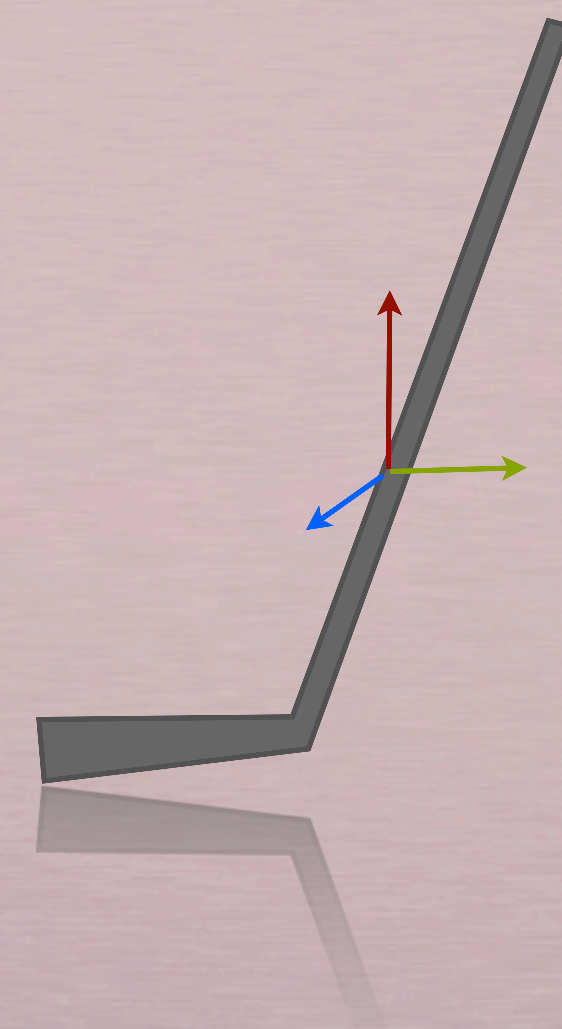
Rigid instruments

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\left(\boxed{\mathbf{q}}, \mathbf{v}\right)$$

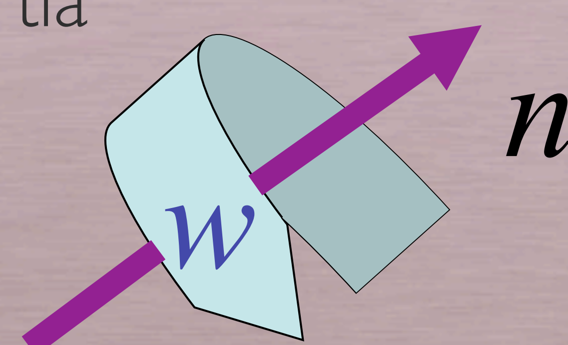
Mass, Inertia

Gravity

Coriolis and centrifugal Forces



- 6 DOFs: 3 translation / 3 rotations => 6 equations
- Motion described at the center of Inertia
- Angular position: use of quaternion



$$q_r = \left| \begin{array}{l} n.\sin(w/2) \\ \cos(w/2) \end{array} \right.$$

MODELING OF INSTRUMENTS

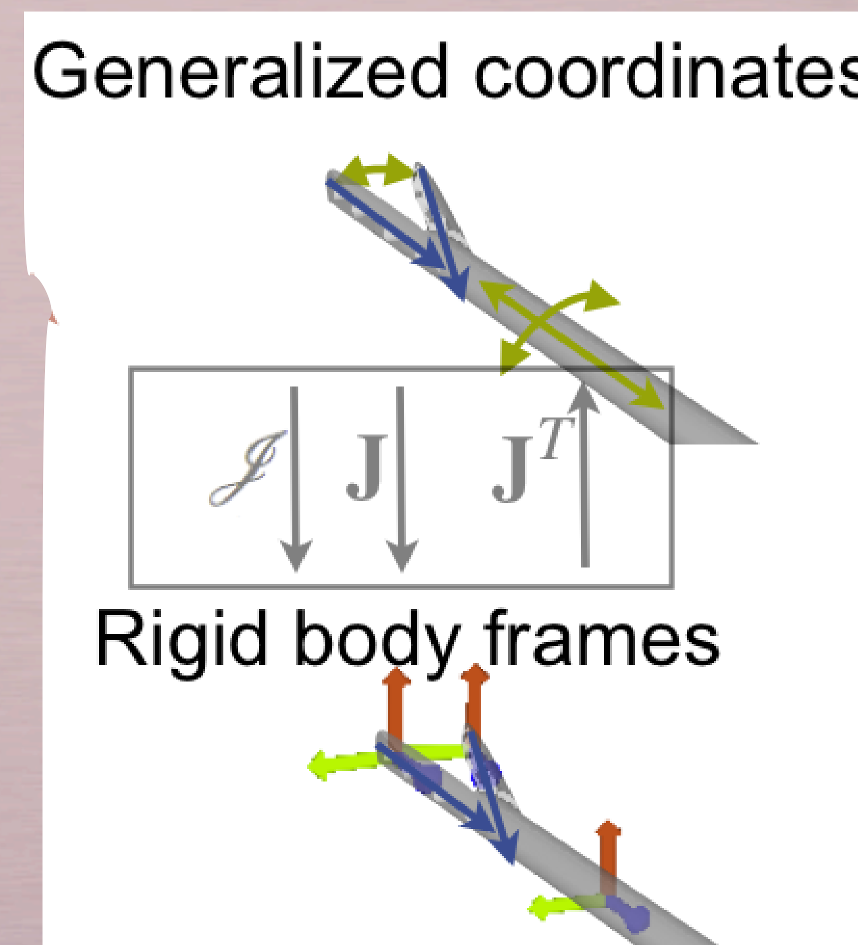
Articulated Models

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\left(\boxed{\mathbf{q}}, \mathbf{v}\right)$$

Mass, Inertia

Gravity

Coriolis and centrifugal Forces



- 6 DOFs + 1 DOF / articulation
- q,v => generalized coordinates (≠ absolute coordinates)
- Tree-like structure

MODELING OF INSTRUMENTS

Wire-like instruments

- Several possible application
  - Interventional radiology instruments: catheters, guides, coils
  - Surgical instruments: flexible needle, suture thread..
- Several possible models
  - Beam Theory (6DoFs per nodes: absolute coords)
  - Cosserat Rods (reduced coordinates)
  - Spline-based models... (it depends...)

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\;(\mathbf{q}, \mathbf{v})$$

- Could also be used for anatomical structures
  - Ligaments
  - Blood vessels

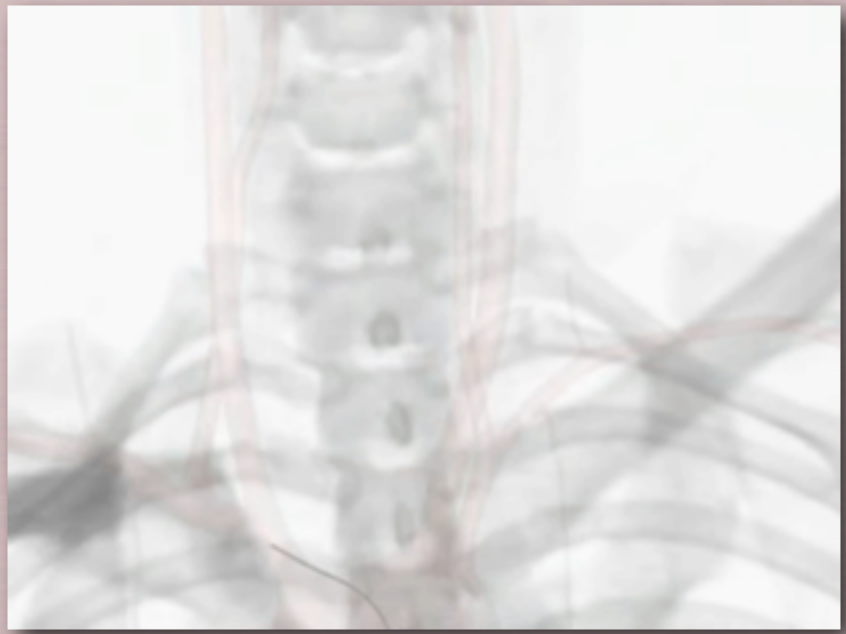
MODELING OF INSTRUMENTS

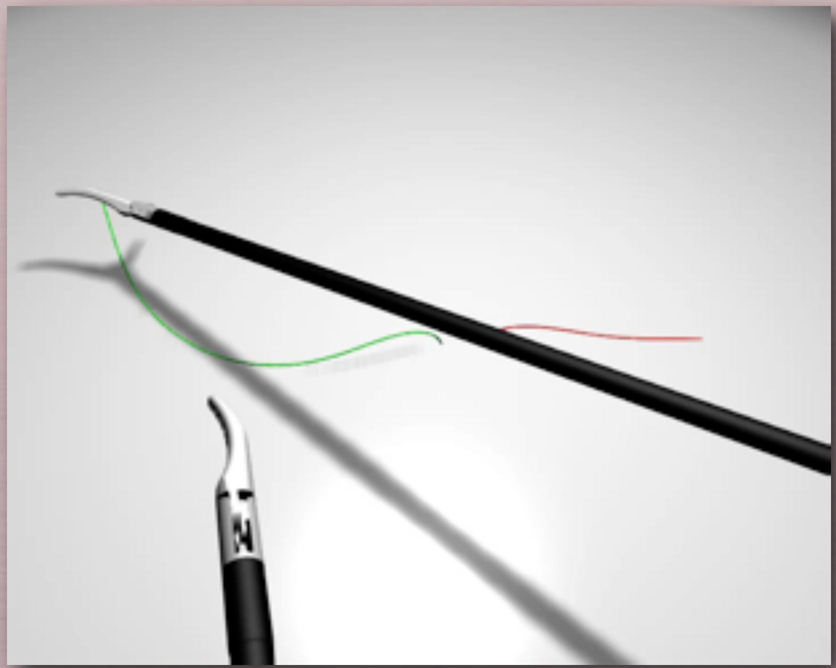
Wire-like instruments

- Several possible application
  - Interventional radiology instruments: catheters, guides, coils
  - Surgical instruments: flexible needle, suture thread..
- Several possible models
  - Beam Theory (6DoFs per nodes: absolute coords)
  - Cosserat Rods (reduced coordinates)
  - Spline-based models... (it depends...)

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\ (\mathbf{q}, \mathbf{v})$$

- Could also be used for anatomical structures
  - Ligaments
  - Blood vessels





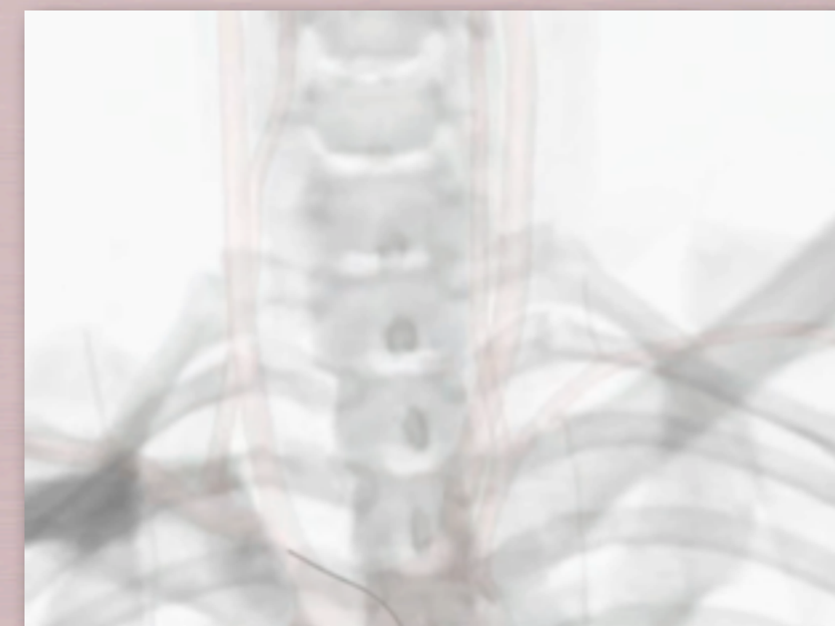
MODELING OF INSTRUMENTS

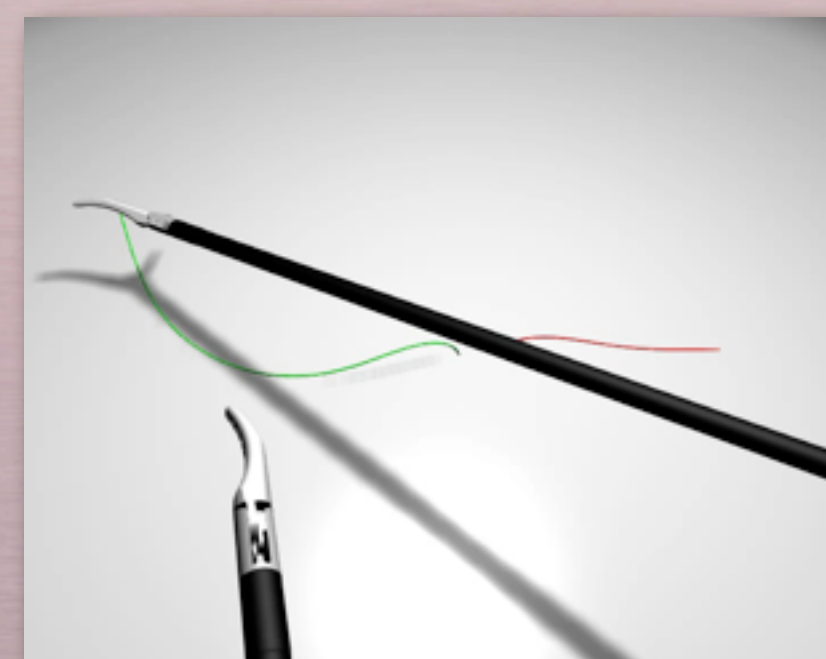
Wire-like instruments

- Several possible application
  - Interventional radiology instruments: catheters, guides, coils
  - Surgical instruments: flexible needle, suture thread..
- Several possible models
  - Beam Theory (6DoFs per nodes: absolute coords)
  - Cosserat Rods (reduced coordinates)
  - Spline-based models... (it depends...)

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\ (\mathbf{q}, \mathbf{v})$$

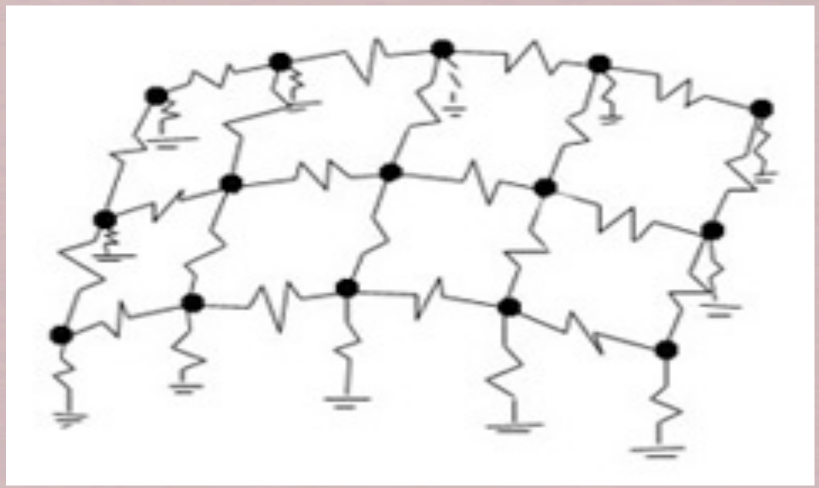
- Could also be used for anatomical structures
  - Ligaments
  - Blood vessels



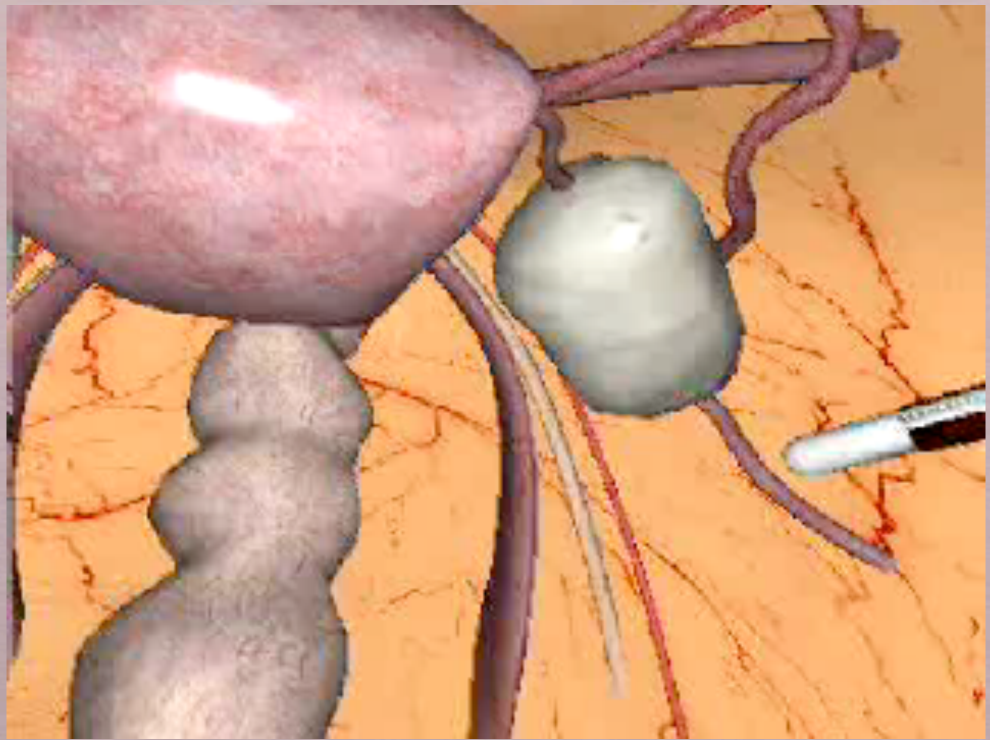


MODELING OF SOFT-TISSUES

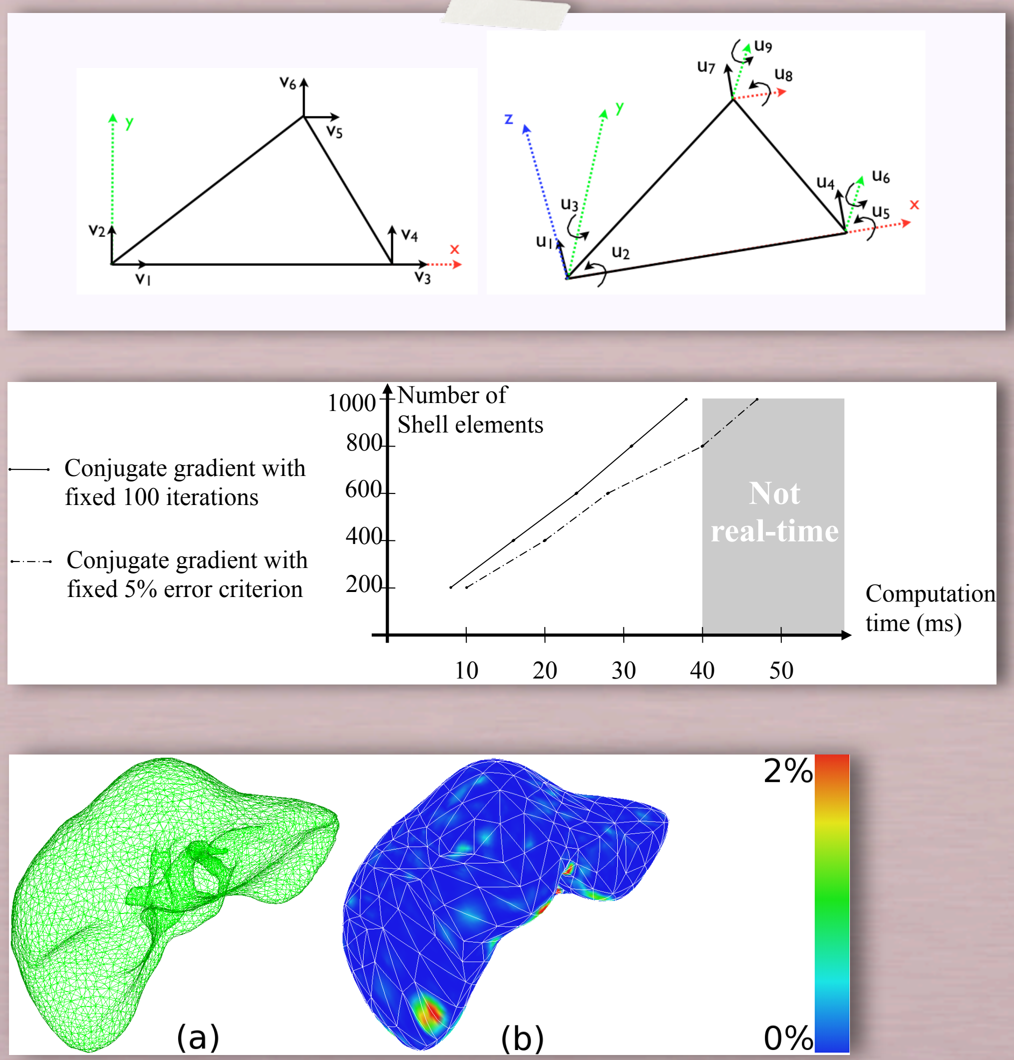
$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\ (\mathbf{q}, \mathbf{v})$$



- Mass-Spring & Discrete models
  - q,v = position and velocity of the nodes
  - Mass lumped at the nodes
  - visco-elastic links
- Positive features
  - Easy to programm !
  - Visual realism
- But not accurate
  - No direct identification & dependance on the mesh



MODELING OF SOFT-TISSUES



- Surface deformations
  - In-Plane (membrane) energy
  - bending energy
- Model
  - Shell theory, mass-springs, patch...

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\ (\mathbf{q}, \mathbf{v})$$

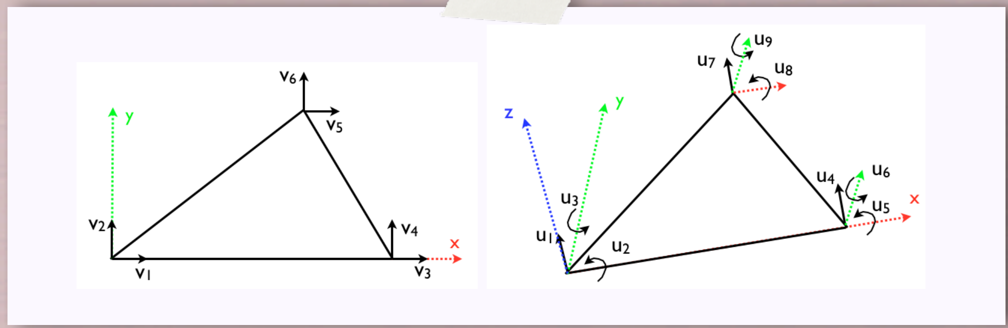
  - q,v (3 to 6 DOFs per nodes...)
- Applications
  - Hollow and tubes organs: bladder, intestine, vessel walls,...
  - Enveloppe: Glisson Capsule (liver), sclera of the eye...
  - Some surgical instruments: lens implant, baloon, ...

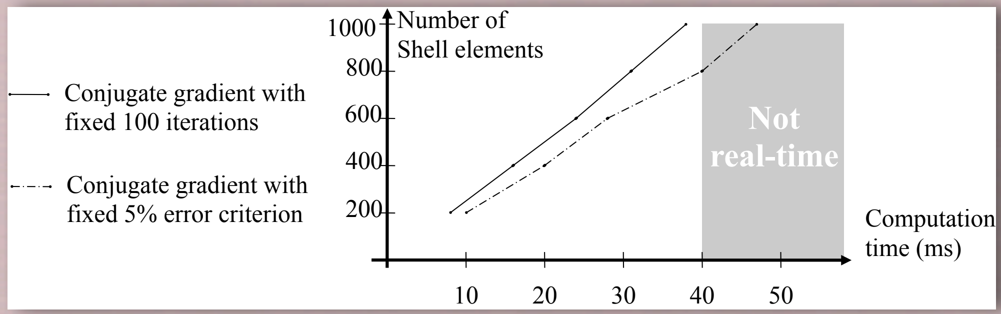
MODELING OF SOFT-TISSUES

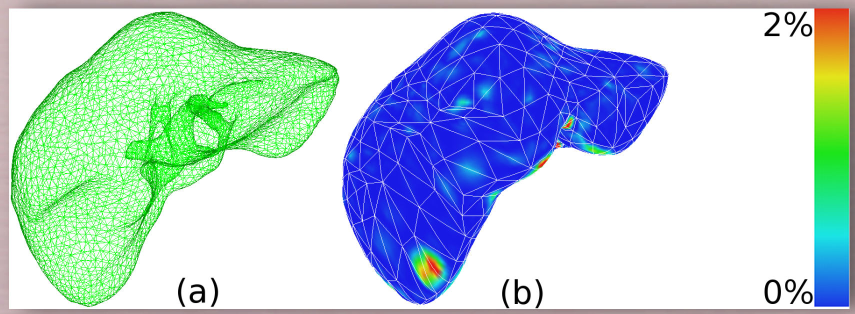
- Surface deformations
  - In-Plane (membrane) energy
  - bending energy
- Model
  - Shell theory, mass-springs, patch...

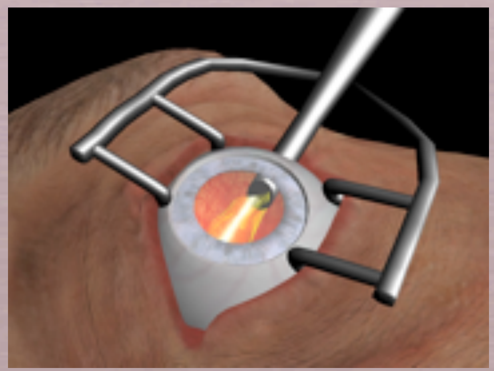
$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\ (\mathbf{q}, \mathbf{v})$$

  - q,v (3 to 6 DOFs per nodes...)
- Applications
  - Hollow and tubes organs: bladder, intestine, vessel walls,...
  - Enveloppe: Glisson Capsule (liver), sclera of the eye...
  - Some surgical instruments: lens implant, baloon, ...



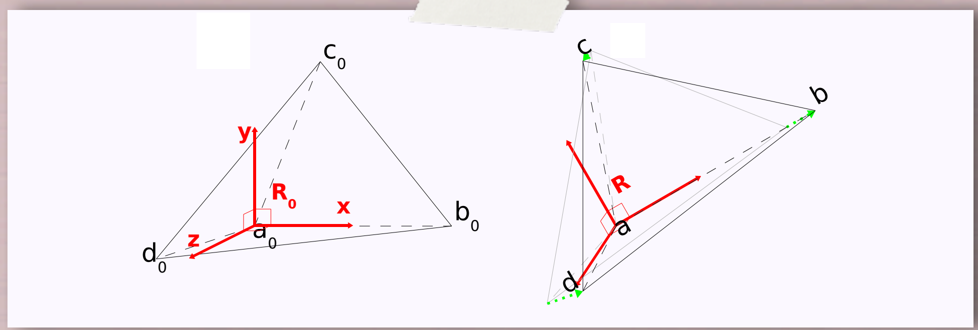




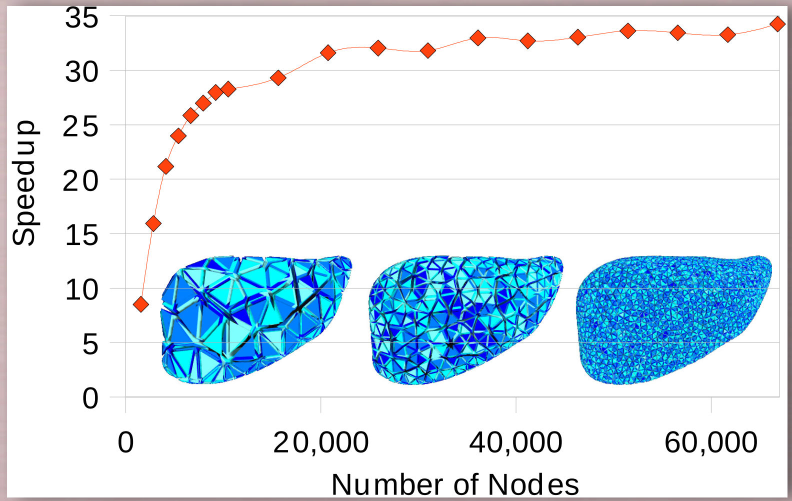


- Mechanical models for real-time computation
- Constraint-based modeling of biomechanical interactions
- Haptic rendering and multithreading approaches
- Applications, ongoing research projects
- Perspective and Conclusion

MODELING OF SOFT-TISSUES



- Volume Deformations
  - No obvious direction with small dimension
  - Mesh (Tetrahedra or Hexahedra)



- FEM Model

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\,(\mathbf{q},\mathbf{v})$$

  - q, v often 3 DoFs per node

- Many application
  - all volume organ, soft-tissues: Parenchyma, muscles, brain-tissues, ... but also bones !

MECHANICAL DEFORMABLE MODELS

- Newton's second law

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\,(\mathbf{q}, \mathbf{v}) + \mathbf{H}^T\lambda$$

$\mathbf{q} \in \mathbb{R}^n$ Vector of generalized degrees of freedom (nodes of a deformable model)

$\mathbf{v} \in \mathbb{R}^n$ Vector of velocities

$\mathbb{M}(\mathbf{q}) : \mathbb{R}^n \mapsto \mathcal{M}^{n\times n}$ Inertia Matrix

$\mathbb{F}\,(\mathbf{q}, \mathbf{v})$ Internal forces (non-linear model)

$\mathbb{P}(t)$ External forces

$\mathbf{H}^T\lambda \in \mathbb{R}^n$ Constraint force contribution

MECHANICAL DEFORMABLE MODELS

- Newton's second law

$$\mathbb{M}(\mathbf{q})\boxed{\dot{\mathbf{v}}} = \mathbb{P}(\boxed{t}) - \mathbb{F}\,(\mathbf{q}, \boxed{\mathbf{v}}) + \mathbf{H}^T\lambda$$

$\mathbf{q} \in \mathbb{R}^n$	Vector of generalized degrees of freedom (nodes of a deformable model)
$\mathbf{v} \in \mathbb{R}^n$	Vector of velocities
$\mathbb{M}(\mathbf{q}) : \mathbb{R}^n \mapsto \mathcal{M}^{n\times n}$	Inertia Matrix
$\mathbb{F}\,(\mathbf{q}, \mathbf{v})$	Internal forces (non-linear model)
$\mathbb{P}(t)$	External forces
$\mathbf{H}^T\lambda \in \mathbb{R}^n$	Constraint force contribution

MECHANICAL DEFORMABLE MODELS

- Newton's second law

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\,(\mathbf{q}, \mathbf{v}) + \mathbf{H}^T\lambda$$

$\mathbf{q} \in \mathbb{R}^n$	Vector of generalized degrees of freedom (nodes of a deformable model)
$\boxed{\mathbf{v} \in \mathbb{R}^n}$	Vector of velocities
$\mathbb{M}(\mathbf{q}) : \mathbb{R}^n \mapsto \mathcal{M}^{n \times n}$	Inertia Matrix
$\boxed{\mathbb{F}\,(\mathbf{q}, \mathbf{v})}$	Internal forces (non-linear model)
$\mathbb{P}(t)$	External forces
$\mathbf{H}^T\lambda \in \mathbb{R}^n$	Constraint force contribution

MECHANICAL DEFORMABLE MODELS

• Newton's second law

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{P}(t) - \mathbb{F}\left(\mathbf{q}, \mathbf{v}\right) + \mathbf{H}^T\lambda$$

$\mathbf{q} \in \mathbb{R}^n$	Vector of generalized degrees of freedom (nodes of a deformable model)
$\boxed{\mathbf{v} \in \mathbb{R}^n}$	Vector of velocities
$\mathbb{M}(\mathbf{q}) : \mathbb{R}^n \mapsto \mathcal{M}^{n \times n}$	Inertia Matrix
$\boxed{\mathbb{F}\left(\mathbf{q}, \mathbf{v}\right)}$	Internal forces (non-linear model)
$\mathbb{P}(t)$	External forces
$\boxed{\mathbf{H}^T\lambda \in \mathbb{R}^n}$	Constraint force contribution

TIME INTEGRATION

- Interactive Simulation = the physician can modify the course of the simulation
- Time derivatives in model equation = integration scheme (notion of time step...)
- Between these two steps, the user do a certain motion in a REAL interval of time
- The time elapsed in the simulation must be EQUAL to the REAL interval of time

TIME INTEGRATION

- Interactive Simulation = the physician can modify the course of the simulation
- Time derivatives in model equation = integration scheme (notion of time step...)
- Between these two steps, the user do a certain motion in a REAL interval of time
- The time elapsed in the simulation must be EQUAL to the REAL interval of time

t

TIME INTEGRATION

- Interactive Simulation = the physician can modify the course of the simulation
- Time derivatives in model equation = integration scheme (notion of time step...)
- Between these two steps, the user do a certain motion in a REAL interval of time
- The time elapsed in the simulation must be EQUAL to the REAL interval of time

t + h

TIME INTEGRATION

- Interactive Simulation = the physician can modify the course of the simulation
- Time derivatives in model equation = integration scheme (notion of time step...)
- Between these two steps, the user do a certain motion in a REAL interval of time
- The time elapsed in the simulation must be EQUAL to the REAL interval of time

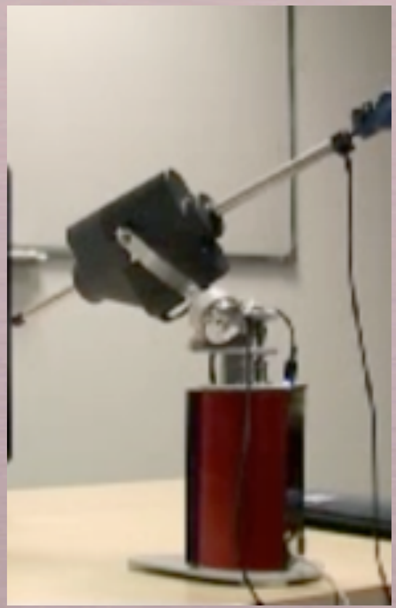
t + h

t

TIME INTEGRATION

- Interactive Simulation = the physician can modify the course of the simulation
- Time derivatives in model equation = integration scheme (notion of time step...)
- Between these two steps, the user do a certain motion in a REAL interval of time
- The time elapsed in the simulation must be EQUAL to the REAL interval of time

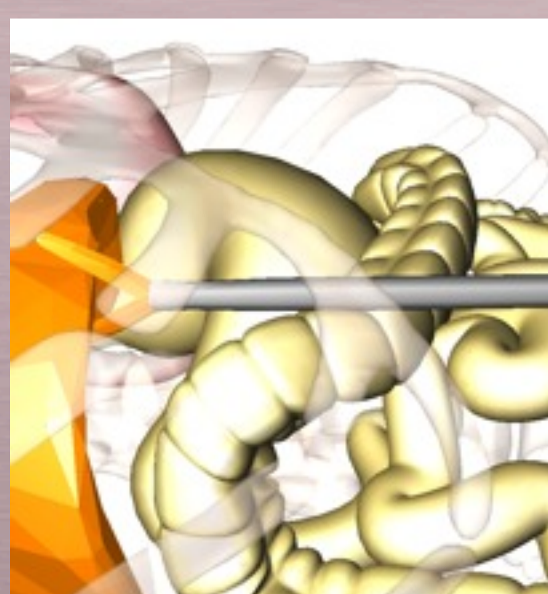
t + h



t + dt

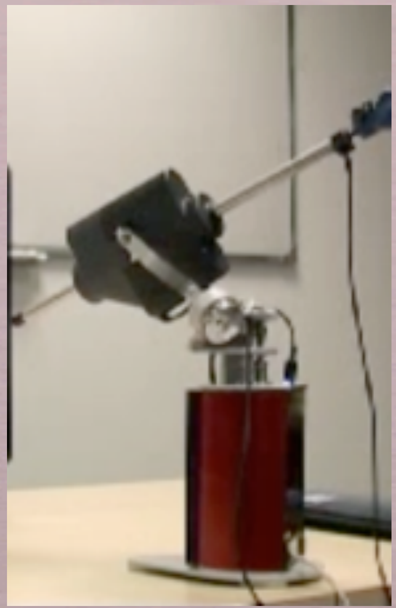
TIME INTEGRATION

- Interactive Simulation = the physician can modify the course of the simulation
- Time derivatives in model equation = integration scheme (notion of time step...)
- Between these two steps, the user do a certain motion in a REAL interval of time
- The time elapsed in the simulation must be EQUAL to the REAL interval of time



TIME INTEGRATION

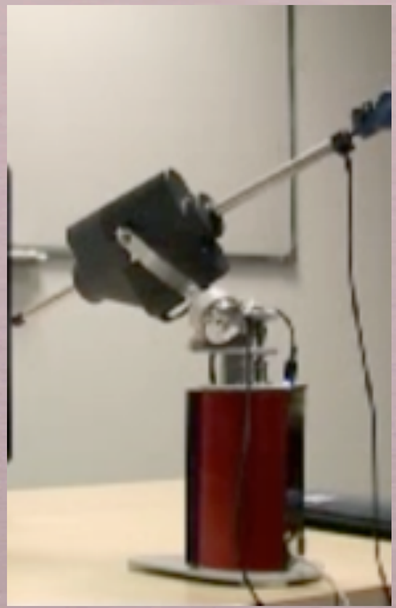
- Interactive Simulation = the physician can modify the course of the simulation
- Time derivatives in model equation = integration scheme (notion of time step...)
- Between these two steps, the user do a certain motion in a REAL interval of time
- The time elapsed in the simulation must be EQUAL to the REAL interval of time



REAL-TIME INTEGRATION

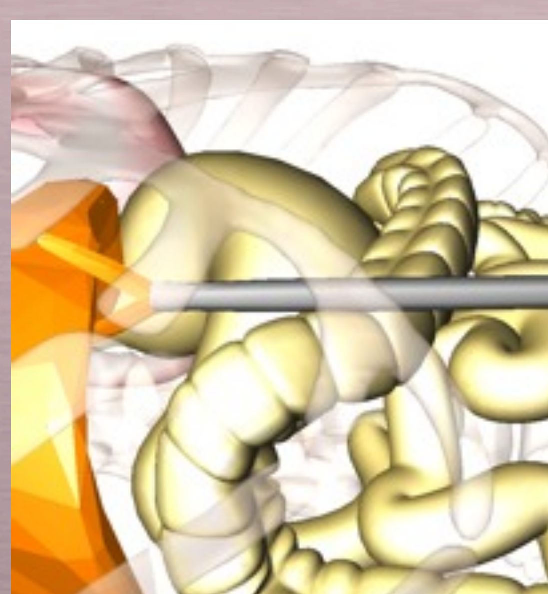
- Interactive Simulation = the physician can modify the course of the simulation
- Time derivatives in model equation = integration scheme (notion of time step...)
- Between these two steps, the user do a certain motion in a REAL interval of time
- The time elapsed in the simulation must be EQUAL to the REAL interval of time

$h = dt$



REAL-TIME INTEGRATION

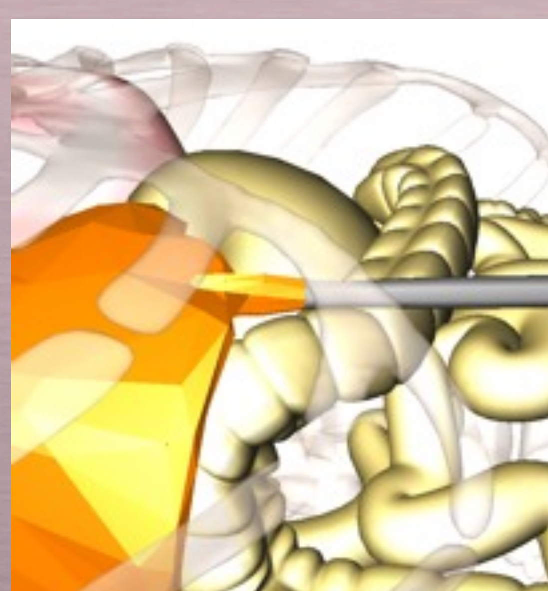
- Interactive Simulation = the physician can modify the course of the simulation
- Time derivatives in model equation = integration scheme (notion of time step...)
- Between these two steps, the user do a certain motion in a REAL interval of time
- The time elapsed in the simulation must be EQUAL to the REAL interval of time



$h = dt$

REAL-TIME INTEGRATION

- Interactive Simulation = the physician can modify the course of the simulation
- Time derivatives in model equation = integration scheme (notion of time step...)
- Between these two steps, the user do a certain motion in a REAL interval of time
- The time elapsed in the simulation must be EQUAL to the REAL interval of time



$$h = dt$$

TIME INTEGRATION SCHEMES

- From Model to Algorithm...:
  - Dynamic equation of 1 point:

$$f(x,v) = Ma$$

  - Euler explicit scheme:

$$v(t) = v(t-dt) + a(t-dt)\, dt$$
$$x(t) = x(t-dt) + v(t-dt)\, dt$$

  - Simulation algorithm

$$t=t+dt \rightarrow \text{integration} \rightarrow f(x(t),v(t)) \rightarrow a(t)=M^{-1}f$$

TIME INTEGRATION SCHEMES

- Explicit Methods:

$$\underbrace{\mathbb{M}\,\dot{\mathbf{v}}}_{\text{unknown}} = \mathbb{P}(t) - \underbrace{\mathbb{F}\,(\mathbf{q},\mathbf{v})}_{\text{known from previous time steps}}$$

  - Conditionnally stable
  - High constraint on the time step used in the simulation
  - **h ≤ Le/c** (**h**: *time step*, **Le**: *Caracterstic lengh of smallest element,* **c**: *velocity of the deformation wave)*

TIME INTEGRATION SCHEMES

- Implicit Methods:

$$\mathbb{M}\,\underbrace{\dot{\mathbf{v}}}_{\text{unknown}} = \mathbb{P}(t) - \mathbb{F}\ \underbrace{(\mathbf{q}, \mathbf{v})}_{\text{depends on } \dot{\mathbf{v}} \text{ and on previous time steps}}$$

  - Unconditionnally stable
  - Possible use of «large» time step **h** in the simulation
  - Needs the resolution of a large non-linear problem

TIME-STEPPING IMPLICIT INTEGRATION

- Implicit Euler Integration
- Use of velocity / impulse formulation

$$\begin{aligned} \mathbf{M}(\mathbf{v}_f - \mathbf{v}_i) &= h\left(\mathbb{P}(t_f) - \mathbb{F}(\mathbf{q}_f, \mathbf{v}_f)\right) \\ \mathbf{q}_f &= \mathbf{q}_i + h\mathbf{v}_f \end{aligned}$$

stability with quite large time-step and «non-smooth» events

- One linearization of the internal forces per time-step

$$\mathbb{F}\left(\mathbf{q}_i + d\mathbf{q}, \mathbf{v}_i + d\mathbf{v}\right) = \mathbf{f}_i + \frac{\delta \mathbb{F}}{\delta \mathbf{q}} d\mathbf{q} + \frac{\delta \mathbb{F}}{\delta \mathbf{v}} d\mathbf{v}$$

(Compromise between precision and computation time)

- A (changing) linear system to be solved at each time step

$$\underbrace{\left(\mathbf{M} + h\frac{\delta \mathbb{F}}{\delta \mathbf{v}} + h^2 \frac{\delta \mathbb{F}}{\delta \mathbf{q}}\right)}_{\mathbf{A}} \underbrace{d\mathbf{v}}_{\mathbf{x}} = \underbrace{-h^2 \frac{\delta \mathbb{F}}{\delta \mathbf{q}} \mathbf{v}_i - h\left(\mathbf{f}_i + \mathbf{p}_f\right)}_{\mathbf{b}}$$

- Switch to quasi-static when computation is too slow

(no notion of «time» in the simulation)

$$\underbrace{\frac{\delta \mathbb{F}}{\delta \mathbf{q}}}_{\mathbf{A}} \underbrace{d\mathbf{q}}_{\mathbf{x}} = \underbrace{\mathbf{P} - \mathbf{f}_{i-1}}_{\mathbf{b}}$$

TIME-STEPPING IMPLICIT INTEGRATION

- Implicit Euler Integration
- Use of velocity / impulse formulation

$$\begin{aligned} \mathbf{M}\boxed{(\mathbf{v}_f - \mathbf{v}_i)} &= \boxed{h}(\mathbb{P}(t_f) - \mathbb{F}(\mathbf{q}_f, \mathbf{v}_f)) \\ \mathbf{q}_f &= \mathbf{q}_i + h\mathbf{v}_f \end{aligned}$$

stability with quite large time-step and «non-smooth» events

- One linearization of the internal forces per time-step

$$\mathbb{F}\left(\mathbf{q}_i + d\mathbf{q}, \mathbf{v}_i + d\mathbf{v}\right) = \mathbf{f}_i + \frac{\delta\mathbb{F}}{\delta\mathbf{q}}d\mathbf{q} + \frac{\delta\mathbb{F}}{\delta\mathbf{v}}d\mathbf{v}$$

(Compromise between precision and computation time)

- A (changing) linear system to be solved at each time step

$$\underbrace{\left(\mathbf{M} + h\frac{\delta\mathbb{F}}{\delta\mathbf{v}} + h^2\frac{\delta\mathbb{F}}{\delta\mathbf{q}}\right)}_{\mathbf{A}} \underbrace{d\mathbf{v}}_{\mathbf{x}} = \underbrace{-h^2\frac{\delta\mathbb{F}}{\delta\mathbf{q}}\mathbf{v}_i - h\left(\mathbf{f}_i + \mathbf{p}_f\right)}_{\mathbf{b}}$$

- Switch to quasi-static when computation is too slow

(no notion of «time» in the simulation)

$$\underbrace{\frac{\delta\mathbb{F}}{\delta\mathbf{q}}}_{\mathbf{A}} \underbrace{d\mathbf{q}}_{\mathbf{x}} = \underbrace{\mathbf{P} - \mathbf{f}_{i-1}}_{\mathbf{b}}$$

TIME-STEPPING IMPLICIT INTEGRATION

- Implicit Euler Integration
- Use of velocity / impulse formulation

$$\begin{aligned}\mathbf{M}(\mathbf{v}_f - \mathbf{v}_i) &= h\left(\mathbb{P}(t_f) - \mathbb{F}(\mathbf{q}_f, \mathbf{v}_f)\right)\\ \mathbf{q}_f &= \mathbf{q}_i + h\mathbf{v}_f\end{aligned}$$

stability with quite large time-step and «non-smooth» events

- One linearization of the internal forces per time-step

$$\mathbb{F}\left(\mathbf{q}_i + d\mathbf{q}, \mathbf{v}_i + d\mathbf{v}\right) = \mathbf{f}_i + \frac{\delta\mathbb{F}}{\delta\mathbf{q}}d\mathbf{q} + \frac{\delta\mathbb{F}}{\delta\mathbf{v}}d\mathbf{v}$$

(Compromise between precision and computation time)

- A (changing) linear system to be solved at each time step

$$\underbrace{\left(\mathbf{M} + h\frac{\delta\mathbb{F}}{\delta\mathbf{v}} + h^2\frac{\delta\mathbb{F}}{\delta\mathbf{q}}\right)}_{\mathbf{A}}\underbrace{d\mathbf{v}}_{\mathbf{x}} = \underbrace{-h^2\frac{\delta\mathbb{F}}{\delta\mathbf{q}}\mathbf{v}_i - h\left(\mathbf{f}_i + \mathbf{p}_f\right)}_{\mathbf{b}}$$

- Switch to quasi-static when computation is too slow

(no notion of «time» in the simulation)

$$\underbrace{\frac{\delta\mathbb{F}}{\delta\mathbf{q}}}_{\mathbf{A}}\underbrace{d\mathbf{q}}_{\mathbf{x}} = \underbrace{\mathbf{P} - \mathbf{f}_{i-1}}_{\mathbf{b}}$$

TIME-STEPPING IMPLICIT INTEGRATION

- Implicit Euler Integration
- Use of velocity / impulse formulation

$$\begin{aligned} \mathbf{M}(\mathbf{v}_f - \mathbf{v}_i) &= h\left(\mathbb{P}(t_f) - \mathbb{F}(\mathbf{q}_f, \mathbf{v}_f)\right) \\ \mathbf{q}_f &= \mathbf{q}_i + h\mathbf{v}_f \end{aligned}$$

stability with quite large time-step and «non-smooth» events

- One linearization of the internal forces per time-step

$$\mathbb{F}\left(\mathbf{q}_i + d\mathbf{q}, \mathbf{v}_i + d\mathbf{v}\right) = \mathbf{f}_i + \frac{\delta\mathbb{F}}{\delta\mathbf{q}}d\mathbf{q} + \frac{\delta\mathbb{F}}{\delta\mathbf{v}}d\mathbf{v}$$

(Compromise between precision and computation time)

- A (changing) linear system to be solved at each time step

$$\underbrace{\left(\mathbf{M} + h\frac{\delta\mathbb{F}}{\delta\mathbf{v}} + h^2\frac{\delta\mathbb{F}}{\delta\mathbf{q}}\right)}_{\boxed{\mathbf{A}}} \underbrace{d\mathbf{v}}_{\boxed{\mathbf{x}}} = \underbrace{-h^2\frac{\delta\mathbb{F}}{\delta\mathbf{q}}\mathbf{v}_i - h\left(\mathbf{f}_i + \mathbf{p}_f\right)}_{\boxed{\mathbf{b}}}$$

- Switch to quasi-static when computation is too slow

(no notion of «time» in the simulation)

$$\underbrace{\frac{\delta\mathbb{F}}{\delta\mathbf{q}}}_{\mathbf{A}} \underbrace{d\mathbf{q}}_{\mathbf{x}} = \underbrace{\mathbf{P} - \mathbf{f}_{i-1}}_{\mathbf{b}}$$

TIME-STEPPING IMPLICIT INTEGRATION

- Implicit Euler Integration
- Use of velocity / impulse formulation

$$\begin{cases} \mathbf{M}(\mathbf{v}_f - \mathbf{v}_i) &= h\left(\mathbb{P}(t_f) - \mathbb{F}(\mathbf{q}_f, \mathbf{v}_f)\right) \\ \mathbf{q}_f &= \mathbf{q}_i + h\mathbf{v}_f \end{cases}$$

stability with quite large time-step and «non-smooth» events

- One linearization of the internal forces per time-step

$$\mathbb{F}\left(\mathbf{q}_i + d\mathbf{q}, \mathbf{v}_i + d\mathbf{v}\right) = \mathbf{f}_i + \frac{\delta \mathbb{F}}{\delta \mathbf{q}} d\mathbf{q} + \frac{\delta \mathbb{F}}{\delta \mathbf{v}} d\mathbf{v}$$

(Compromise between precision and computation time)

- A (changing) linear system to be solved at each time step

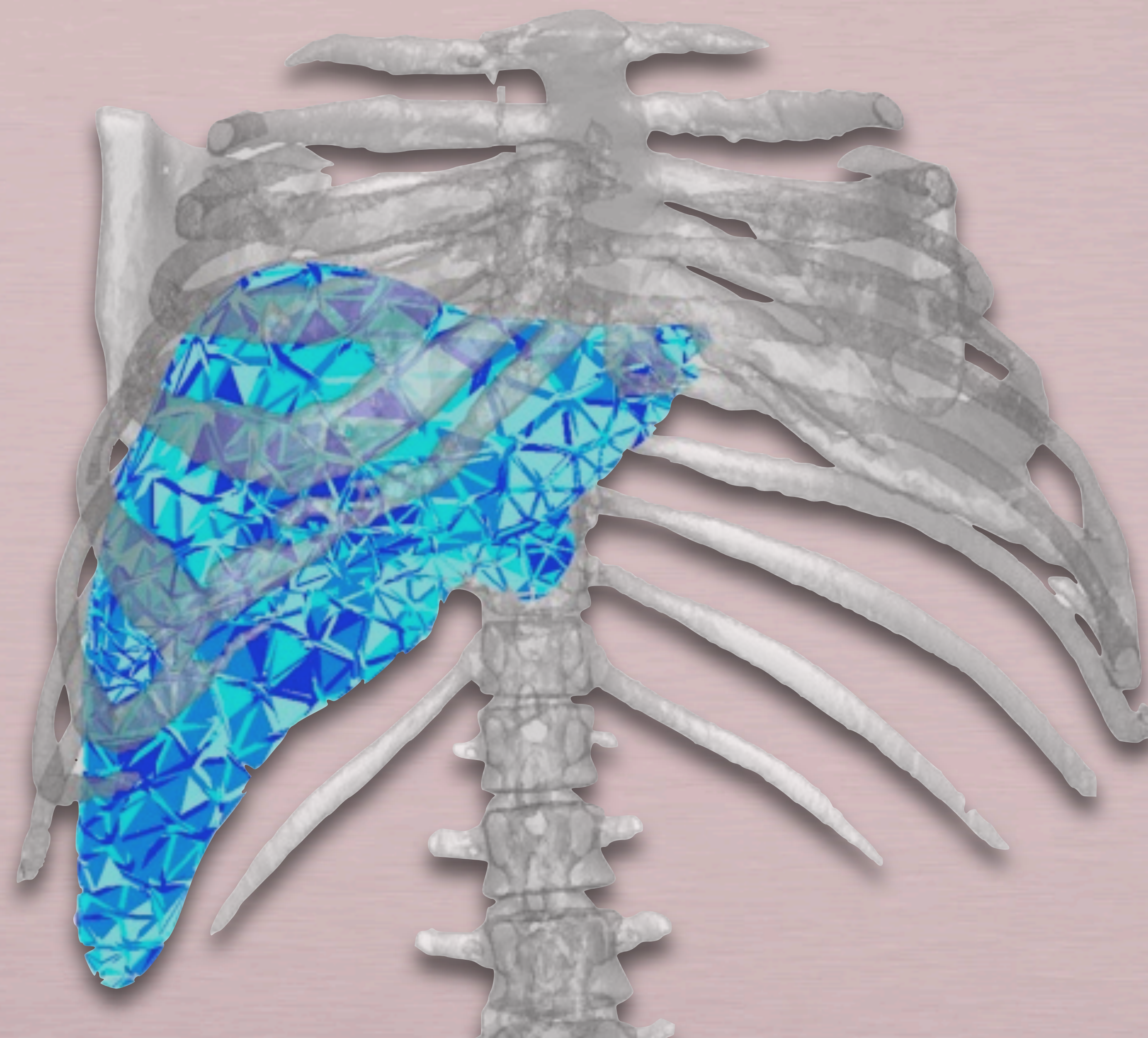
$$\underbrace{\left(\mathbf{M} + h\frac{\delta \mathbb{F}}{\delta \mathbf{v}} + h^2\frac{\delta \mathbb{F}}{\delta \mathbf{q}}\right)}_{\boxed{\mathbf{A}}} \underbrace{d\mathbf{v}}_{\boxed{\mathbf{x}}} = \underbrace{-h^2\frac{\delta \mathbb{F}}{\delta \mathbf{q}}\mathbf{v}_i - h\left(\mathbf{f}_i + \mathbf{p}_f\right)}_{\boxed{\mathbf{b}}}$$

- Switch to quasi-static when computation is too slow

(no notion of «time» in the simulation)

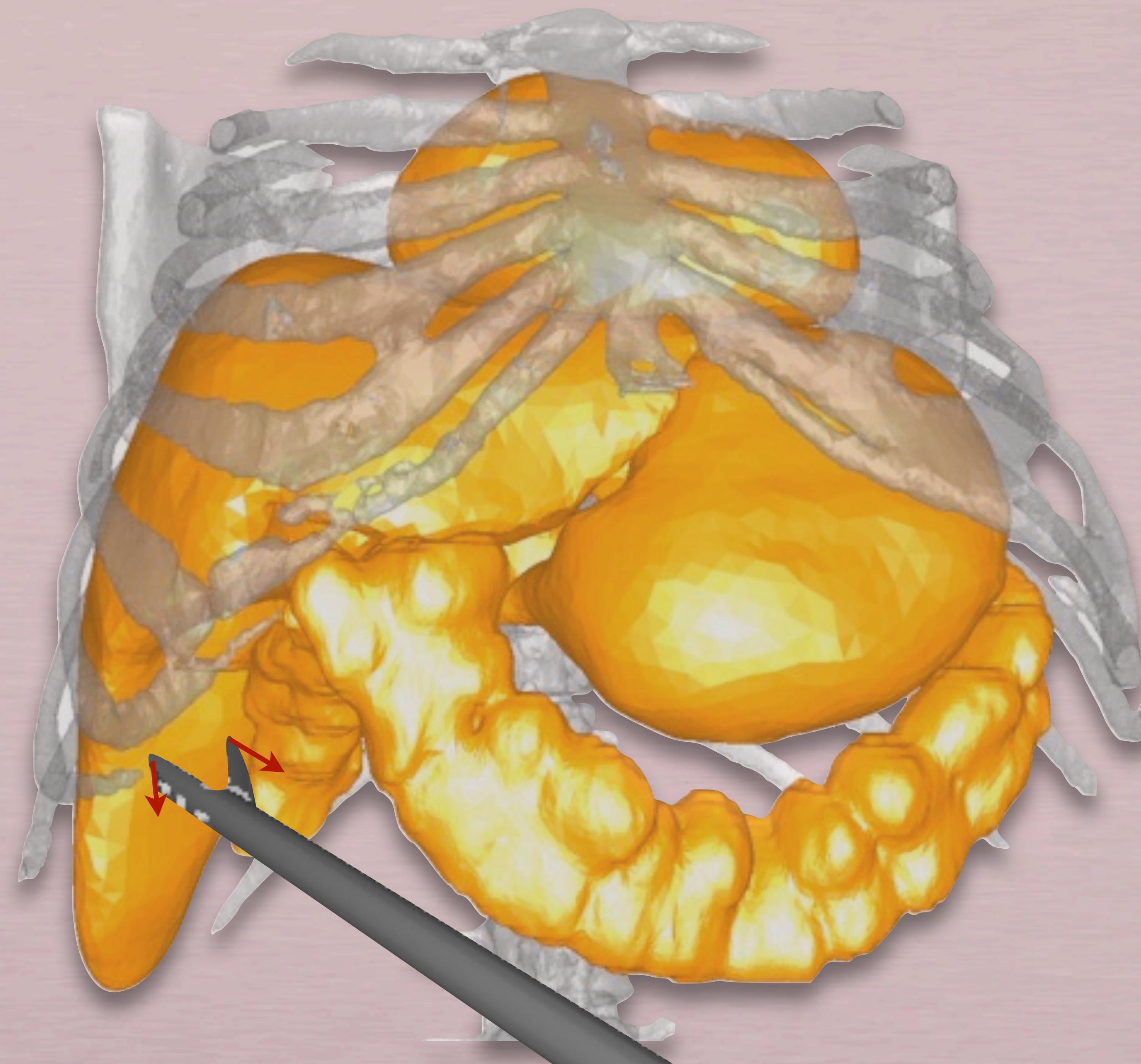
$$\underbrace{\frac{\delta \mathbb{F}}{\delta \mathbf{q}}}_{\boxed{\mathbf{A}}} \underbrace{d\mathbf{q}}_{\boxed{\mathbf{x}}} = \underbrace{\mathbf{P} - \mathbf{f}_{i-1}}_{\boxed{\mathbf{b}}}$$

- Mechanical deformable models for real-time computation
- Constraint-based modeling of biomechanical interactions
- Haptic rendering and multithreading approaches
- Applications, ongoing research projects
- Perspective and Conclusion



CONSTRAINT-BASED MODELING OF BIOMECHANICAL INTERACTIONS

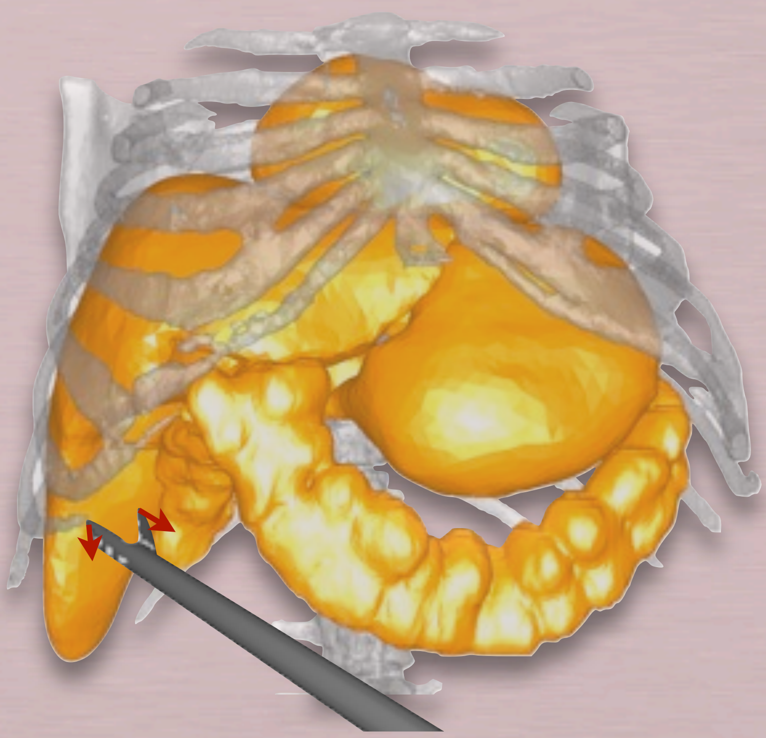
- Constraint-based modeling of biomechanical interactions
- Haptic rendering and multithreading approaches
- Applications, ongoing research projects
- Perspective and Conclusion

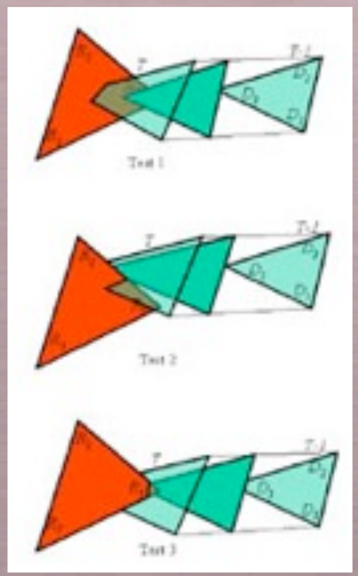


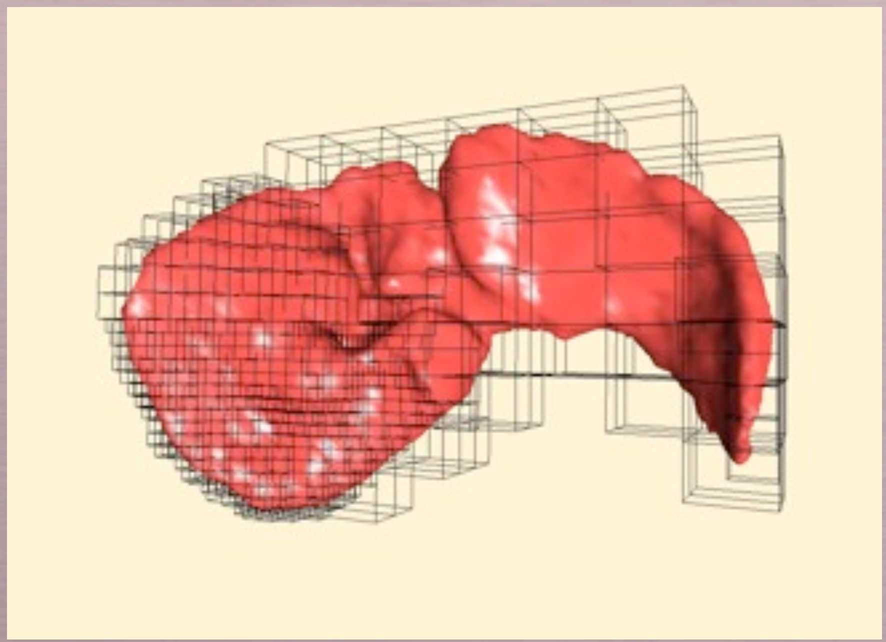
CONSTRAINT-BASED MODELING OF BIOMECHANICAL INTERACTIONS

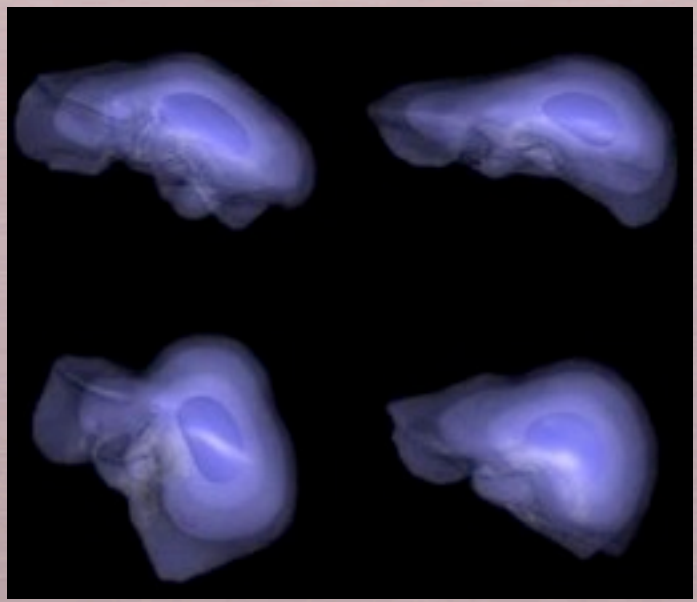
COLLISION DETECTION

- A simple problem: find the colliding primitives of two meshes...
  - ...that requires many computation !!!
  - Various optimization:
    - Bounding Volume hierarchy
    - Spatial coherency
    - Distance maps, implicit representation...

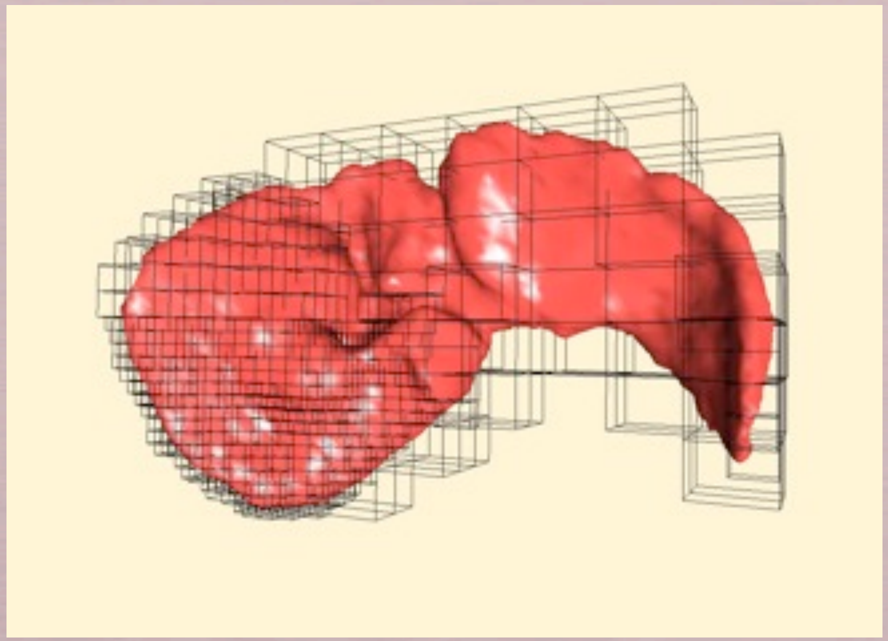








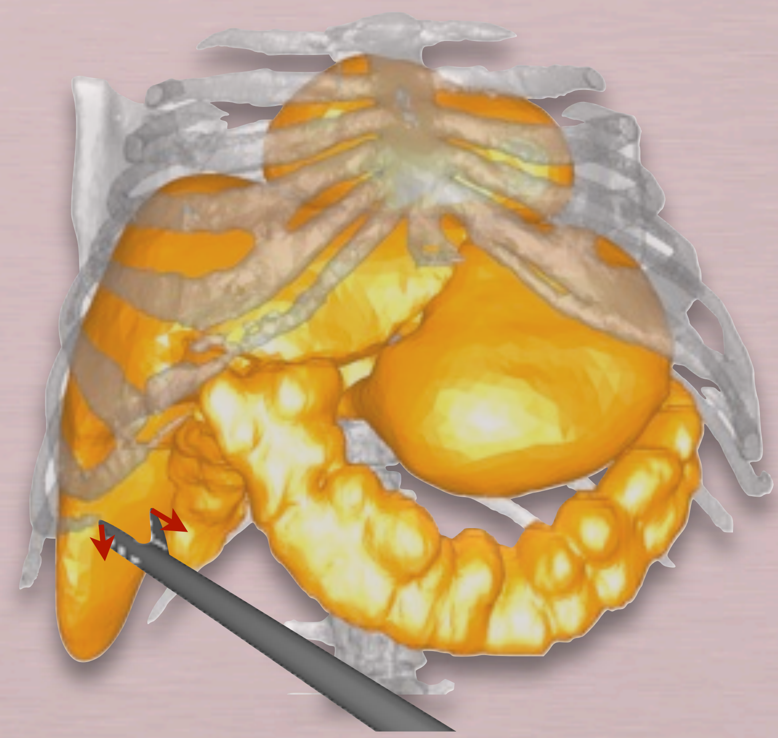
COLLISION DETECTION



NON-SMOOTH (BIO)-MECHANICS IN REAL-TIME

- Why important ?
  - Boundary conditions between anatomical structures
  - Device-tissues interactions
- Why difficult ?
  - Non-smooth events
  - Multi-Contact response
    - Contact: Signorini's law (linear inequalities)
    - Friction: Coulomb's law (non-linear inequalities)
  - Many other interactions...
    - Complex anatomical and mechanical links between organs
    - Specific interactions for some devices

$$\begin{aligned} \mathbf{M}(\mathbf{v}_f - \mathbf{v}_i) &= h\left(\mathbb{P}(t_f) - \mathbb{F}(\mathbf{q}_f, \mathbf{v}_f)\right) + \ h\mathbf{H}^T\lambda_f \\ \mathbf{q}_f &= \mathbf{q}_i + h\mathbf{v}_f \end{aligned}$$

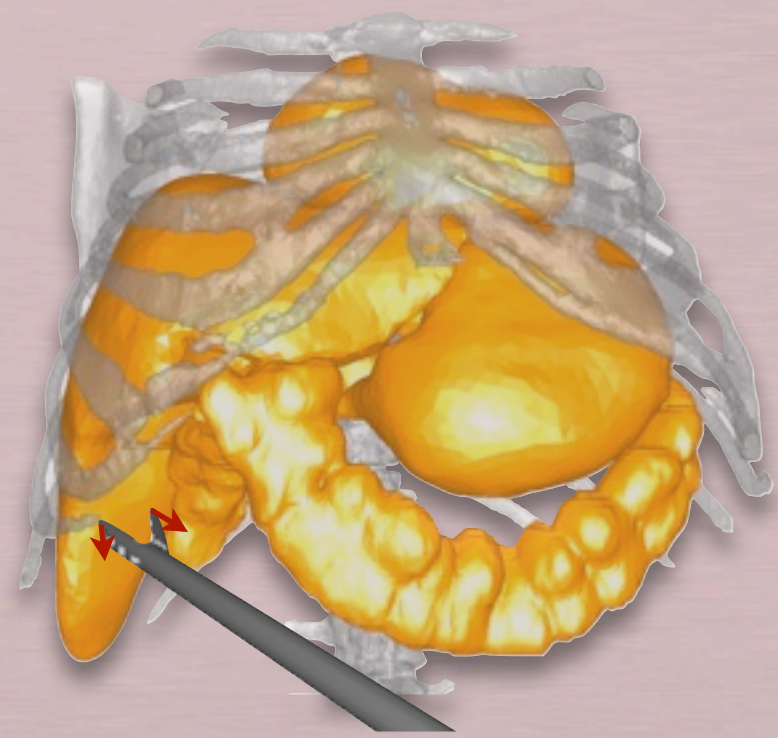


Example : V- <0 before impact and V+ >0 after impact.. between them, an very small time step

NON-SMOOTH (BIO)-MECHANICS IN REAL-TIME

- Why important ?
  - Boundary conditions between anatomical structures
  - Device-tissues interactions
- Why difficult ?
  - Non-smooth events
  - Multi-Contact response
    - Contact: Signorini's law (linear inequalities)
    - Friction: Coulomb's law (non-linear inequalities)
  - Many other interactions...
    - Complex anatomical and mechanical links between organs
    - Specific interactions for some devices

$$\begin{aligned}\mathbf{M}(\mathbf{v}_f - \mathbf{v}_i) &= h\left(\mathbb{P}(t_f) - \mathbb{F}(\mathbf{q}_f, \mathbf{v}_f)\right) + \boxed{h\mathbf{H}^T\lambda_f} \\ \mathbf{q}_f &= \mathbf{q}_i + h\mathbf{v}_f\end{aligned}$$



Example : V- <0 before impact and V+ >0 after impact.. between them, an very small time step

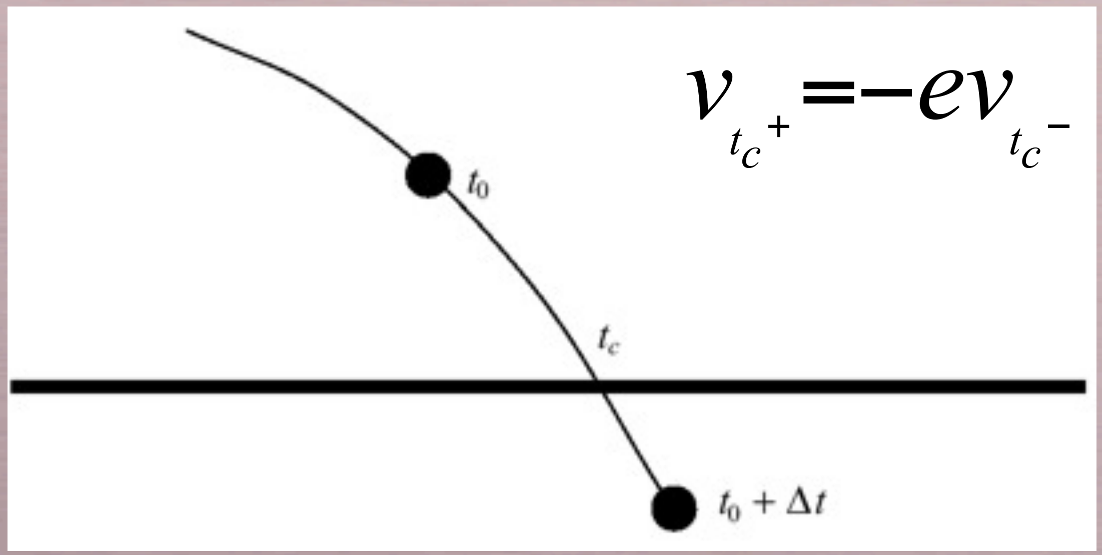
EVENT-DRIVEN

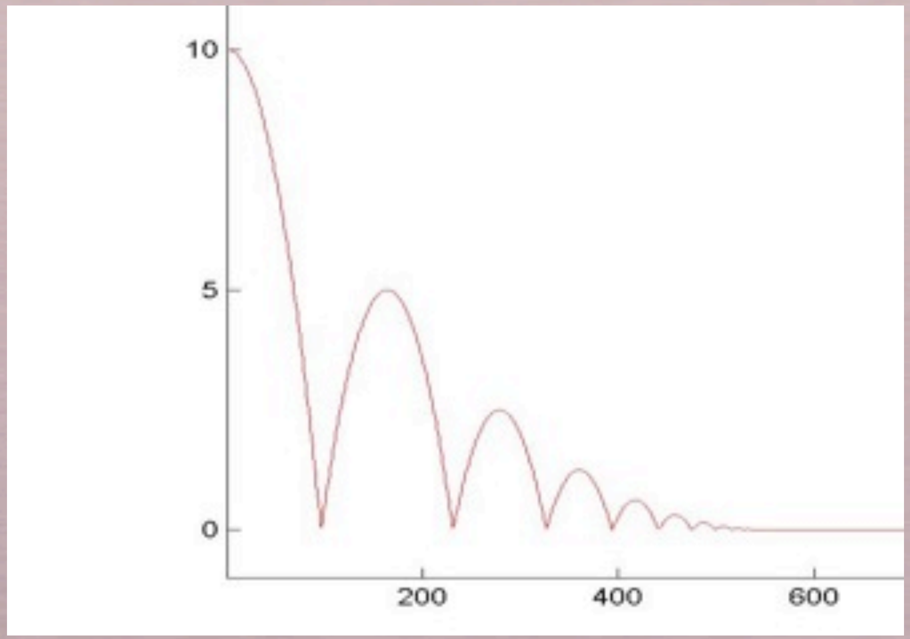
- A chaque nouvelle collision:
  - On détermine l'instant du premier impact,
  - On arrête l'intégration en temps,
  - On résout l'impact,
  - On redémarre l'intégration en temps

- Intérêt:
  - Entre deux évènements, on respecte la continuité nécessaire aux schémas d'intégration « élevés », donc précis.
- Défaut:
  - Coûteux en résolution si on a beaucoup de contacts,
  - Rebond « infini »… on est obligé de donner un critère d'arrêt

EVENT-DRIVEN

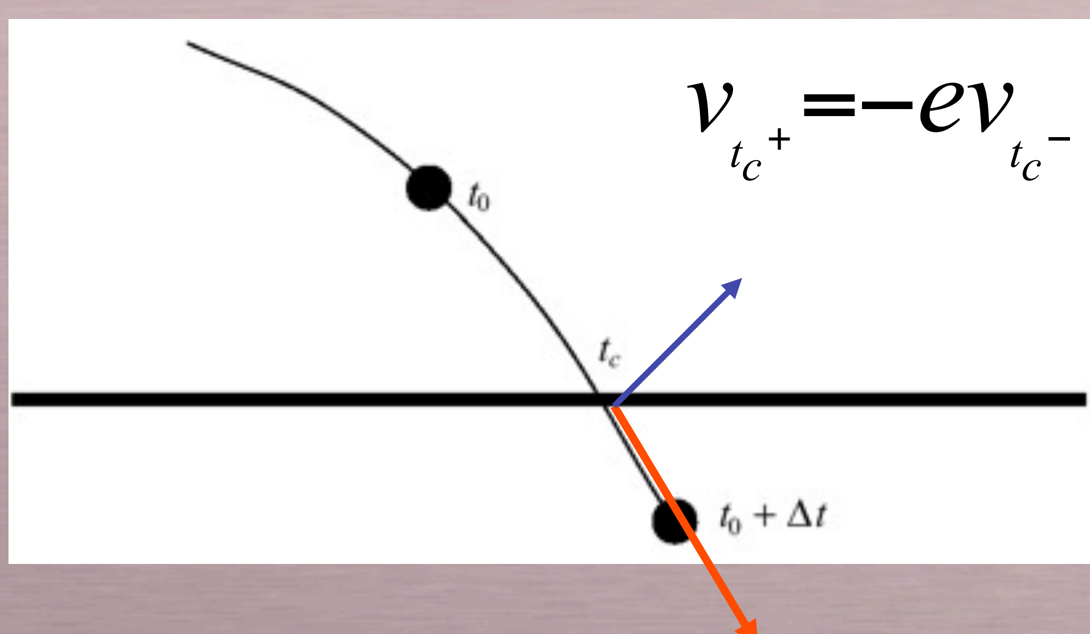
- Collision contre un mur
  - Temps de collision tc
  - Facteur de restitution e de Newton
  - Problème du rebond à l'infini

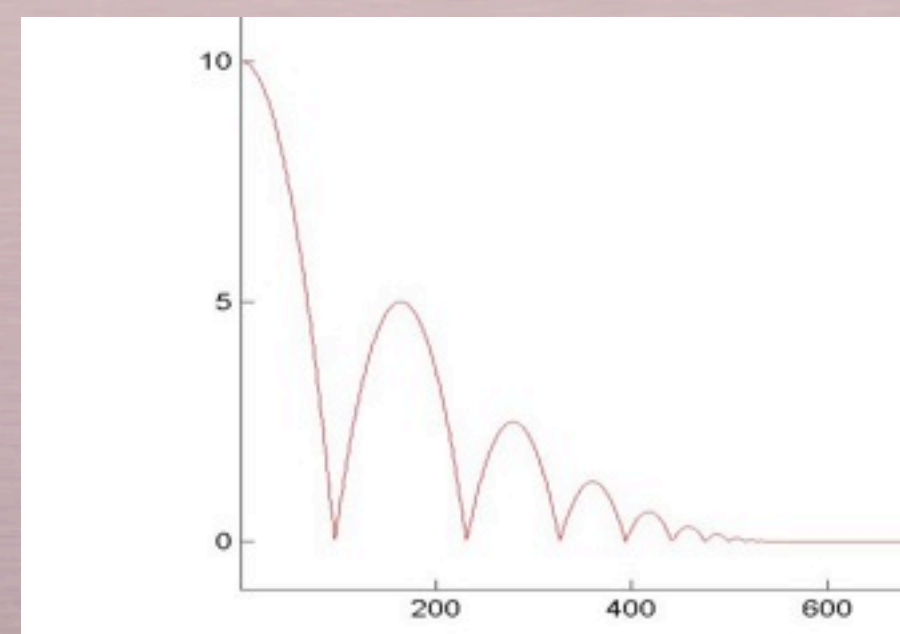




EVENT-DRIVEN

- Collision contre un mur
  - Temps de collision tc
  - Facteur de restitution e de Newton
  - Problème du rebond à l'infini



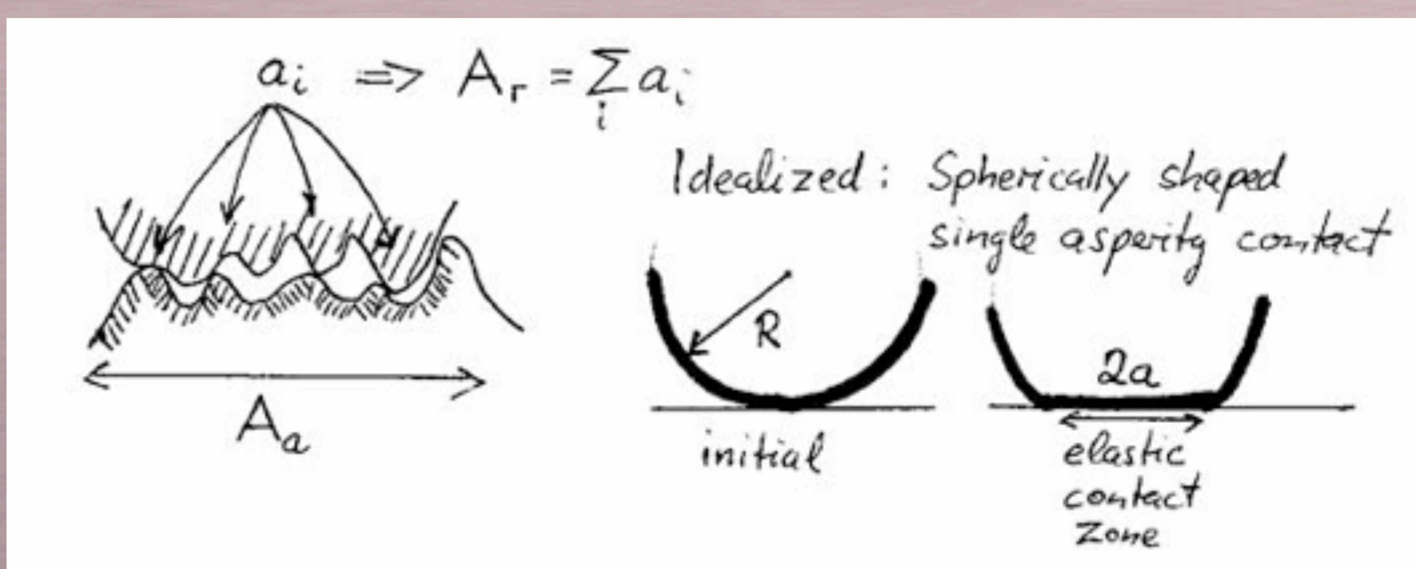


TIME-STEPPING

- Pas de temps FIXE
  - On détecte toutes les collisions apparues durant le pas de temps,
  - On résout toutes ces collisions « en même temps »,
  - Mouvement « contraint » avec toutes les forces
- Intérêt:
  - Plus rapide si il y a beaucoup de contacts,
- Défaut :
  - Utilisation de schéma d'intégration d'ordre faible
    (Forces de contact / impact deviennent des impulsions)
  - Pas de temps d'intégration petits

CONTACT'S LAW

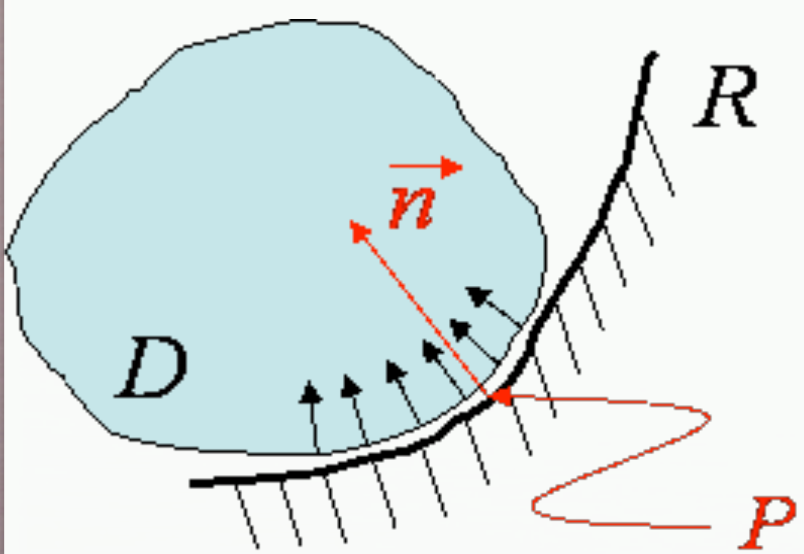
- Hertz model:
  - the strains are small and within the elastic limit,
  - each body can be considered an elastic half-space, i.e., the area of contact is much smaller than the characteristic radius of the body,
  - the surfaces are continuous and non-conforming
  - the surfaces are frictionless.
  - **analytical solutions**



$$a^3 = \frac{3PR}{4E^*}; \quad E^* = \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}\right); \quad a \ll R$$

CONTACT'S LAW

- Signorini's law
  - Complementarity
  - Contact betwen soft object and its environment



Non pénétration : $\delta_n(P) \geq 0$

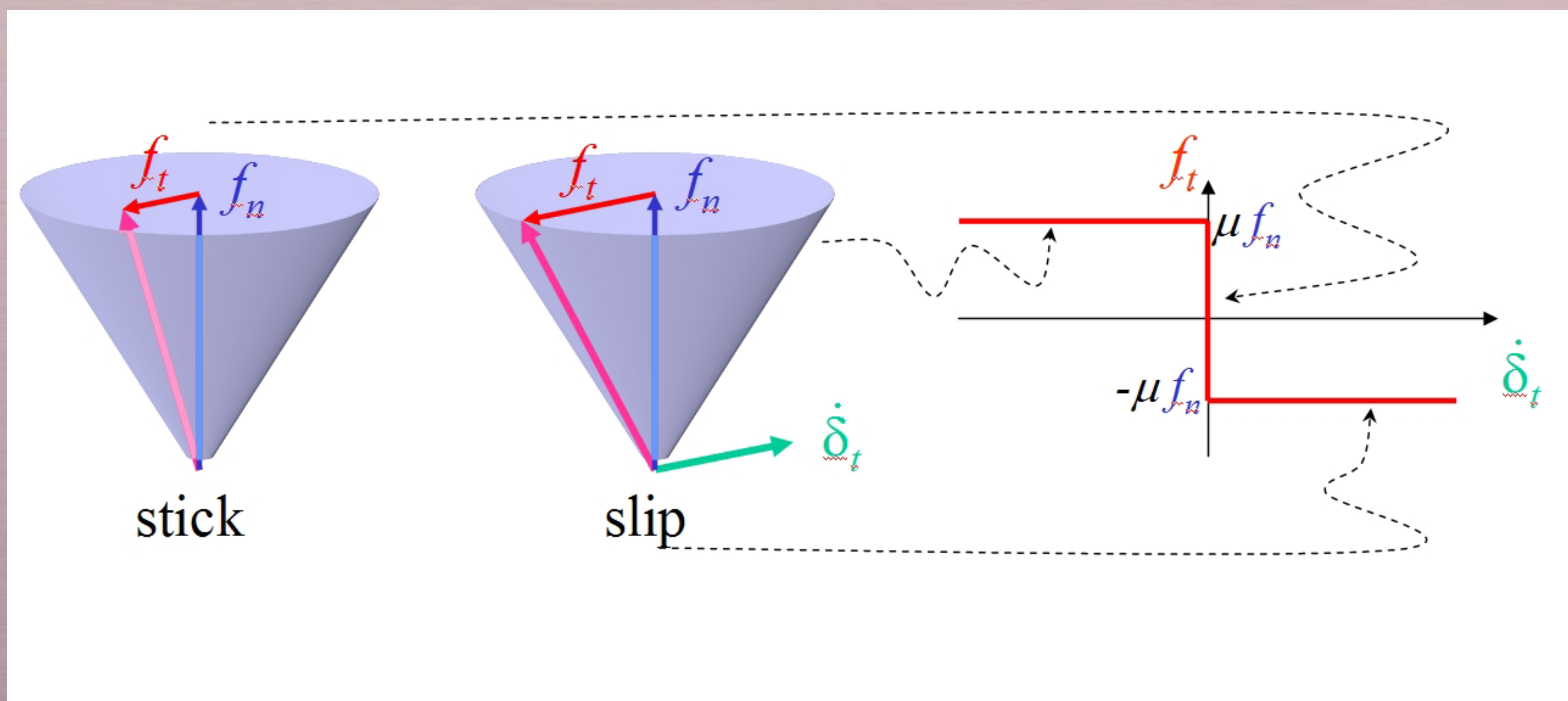
Pression à la surface : $\sigma_{nn}(P) \geq 0$

Complémentarité : $\delta_n(P) \perp \sigma_{nn}(P)$

FRICTION'S LAW

- Coulomb's law
  - Complementarity too...

$$\dot{\delta}_{\vec{T}} = \vec{0} \Rightarrow \|f_{\vec{T}}\| < \mu \, \|f_{\vec{n}}\| \quad \text{(stick)}$$
$$\dot{\delta}_{\vec{T}} \neq \vec{0} \Rightarrow f_{\vec{T}} = -\mu \, \|f_{\vec{n}}\| \frac{\dot{\delta}_{\vec{T}}}{\|\dot{\delta}_{\vec{T}}\|} = -\mu \, \|f_{\vec{n}}\| \, \vec{T} \quad \text{(slip)}$$



«CONTACT MAPPING»

- How to build matrix $\mathbf{H}^T$?
  - Direction of the contact $\mathfrak{F}_\alpha \doteq [\mathbf{n}_\alpha, \mathbf{t}_\alpha, \mathbf{s}_\alpha].$
  - Link between constraint motion and DOFs

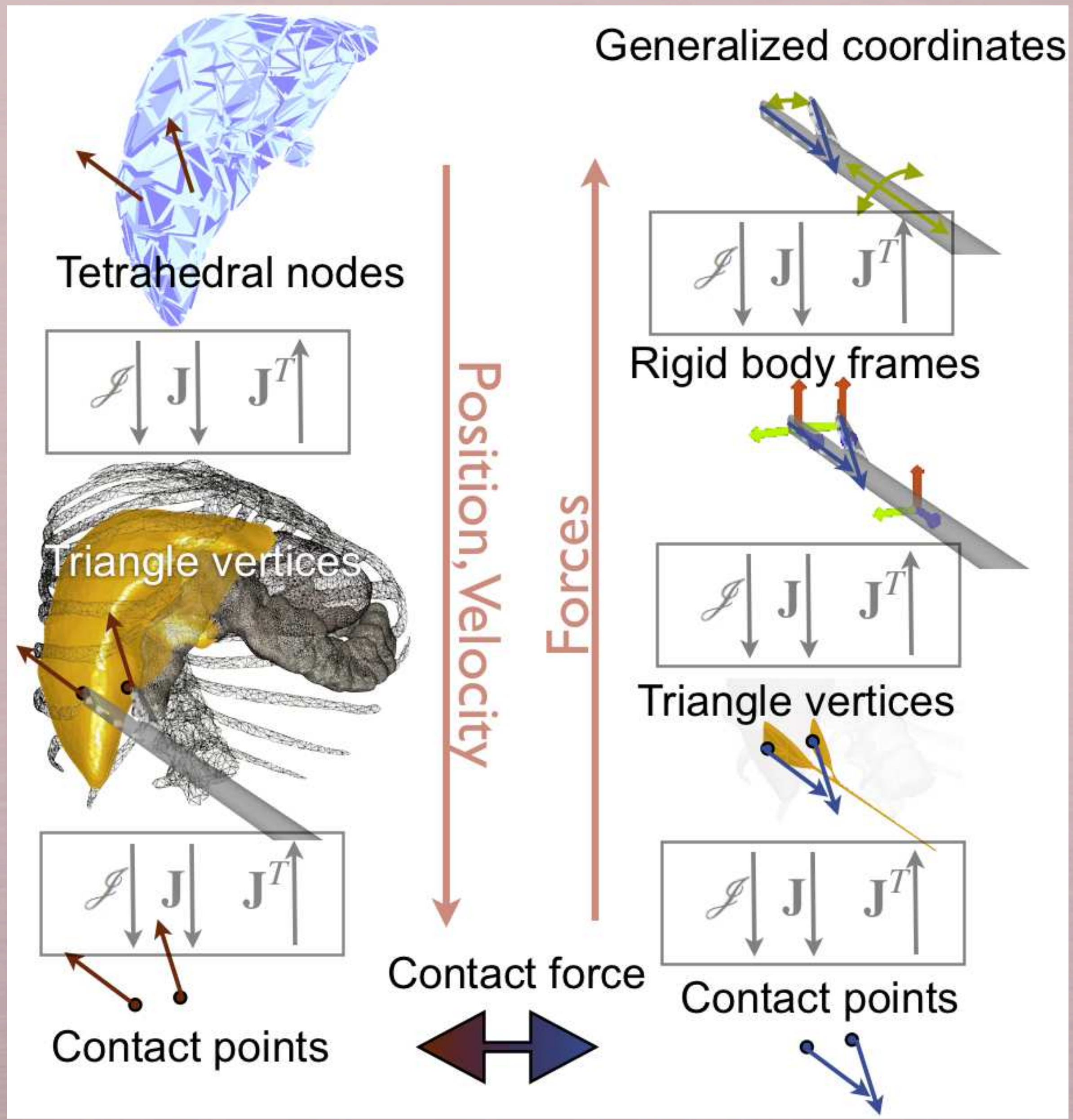
$$\boldsymbol{\delta}_\alpha = \mathbb{A}_\alpha(\mathbf{q_1}, t) - \mathbb{A}_\alpha(\mathbf{q_2}, t)$$

  - Derivation

$$\mathbb{H}_\alpha(\mathbf{q}) = \frac{\partial \mathbb{A}_\alpha}{\partial \mathbf{q}}$$

$$\dot{\boldsymbol{\delta}}_\alpha(t) = \mathbb{H}_\alpha(\mathbf{q_1})\mathbf{v}_1(t) - \mathbb{H}_\alpha(\mathbf{q_2})\mathbf{v}_2(t)$$

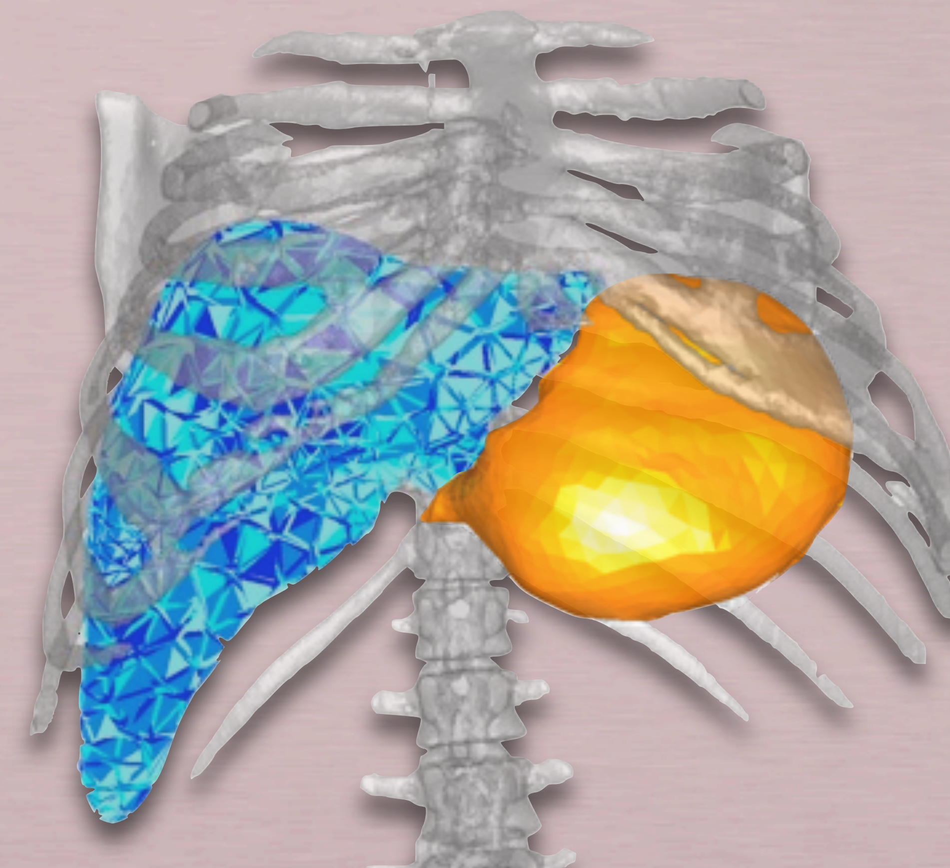
  - Virtual work principle
    - Force = $\mathbb{H}(\mathbf{q_i})^T \boldsymbol{\lambda}_f$



INTERACTION BETWEEN TWO DEFORMABLE MODELS

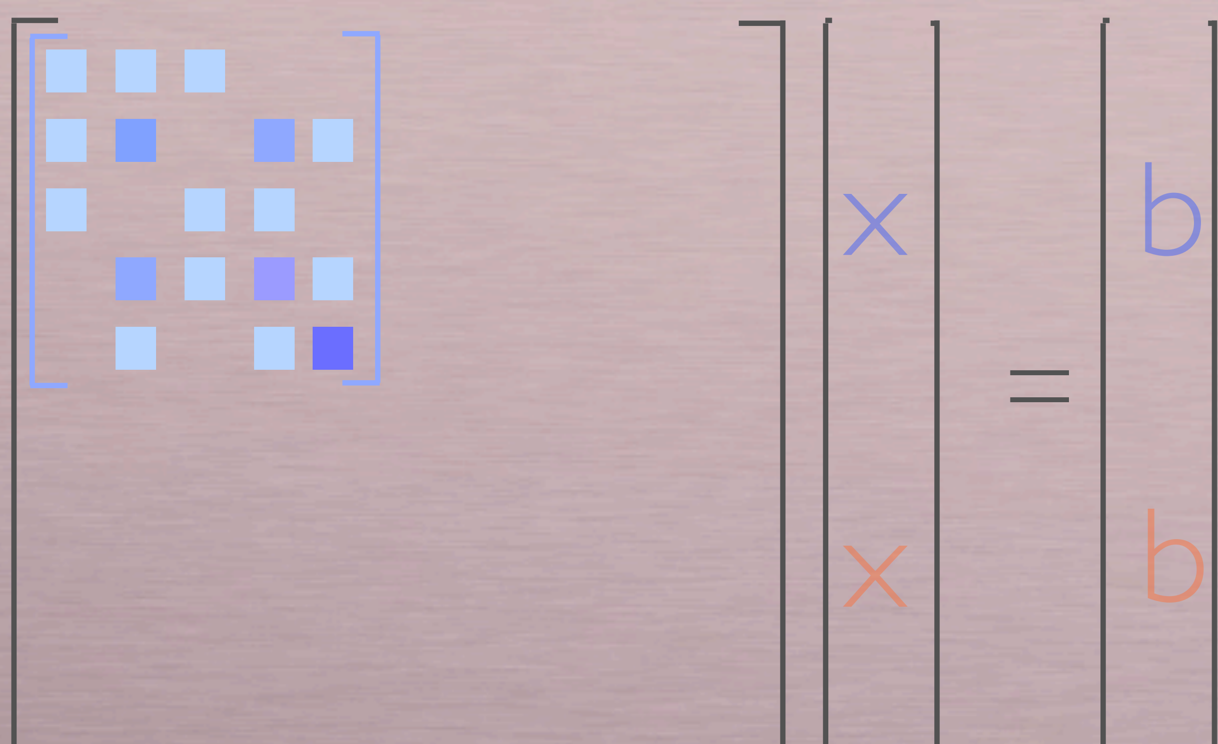
- 2 system of equations A x = b for each object
- «Direct approach»
  - Penalty forces: additional stiffness in the system
  - Lagrange Multipliers: very large system of (in)-equations

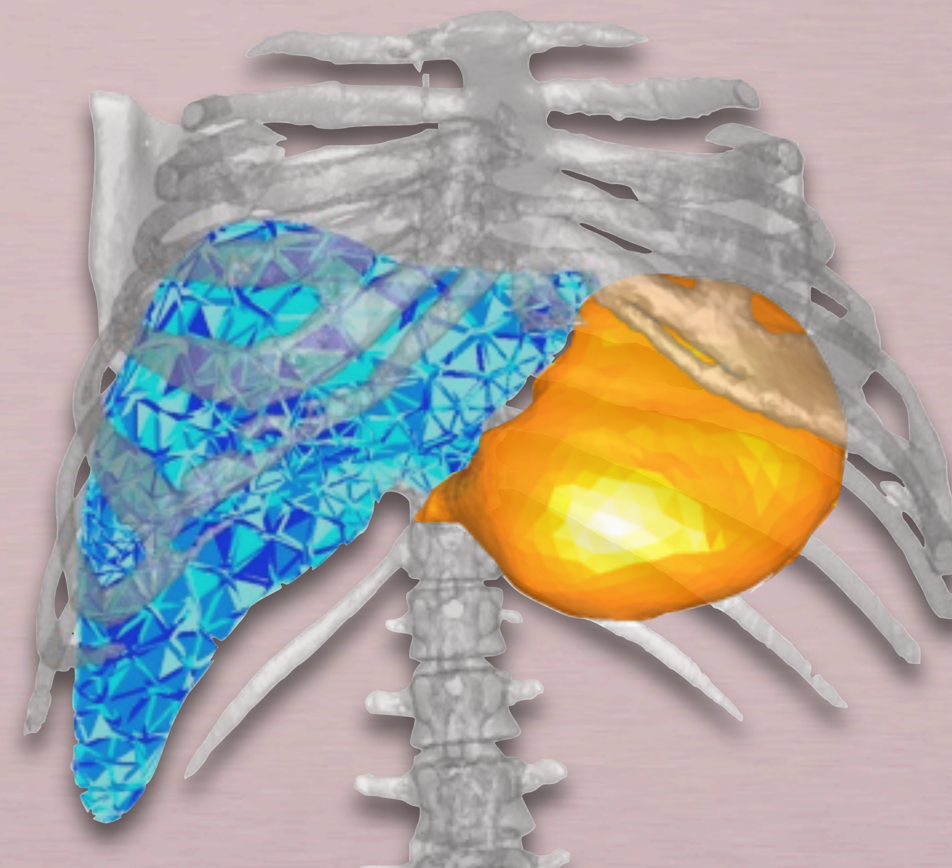
$$\begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}\begin{bmatrix} x \\ x \end{bmatrix} = \begin{bmatrix} b \\ b \end{bmatrix}$$



INTERACTION BETWEEN TWO DEFORMABLE MODELS

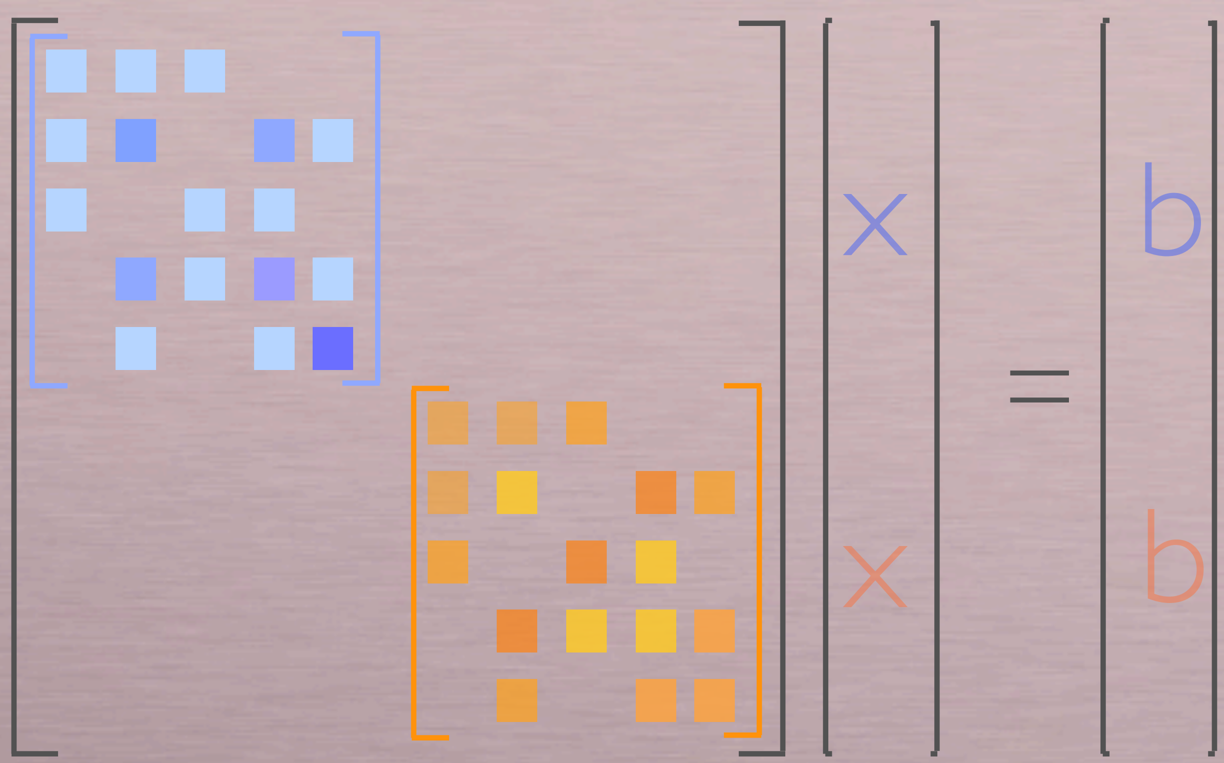
- 2 system of equations A x = b for each object
- «Direct approach»
  - Penalty forces: additional stiffness in the system
  - Lagrange Multipliers: very large system of (in)-equations

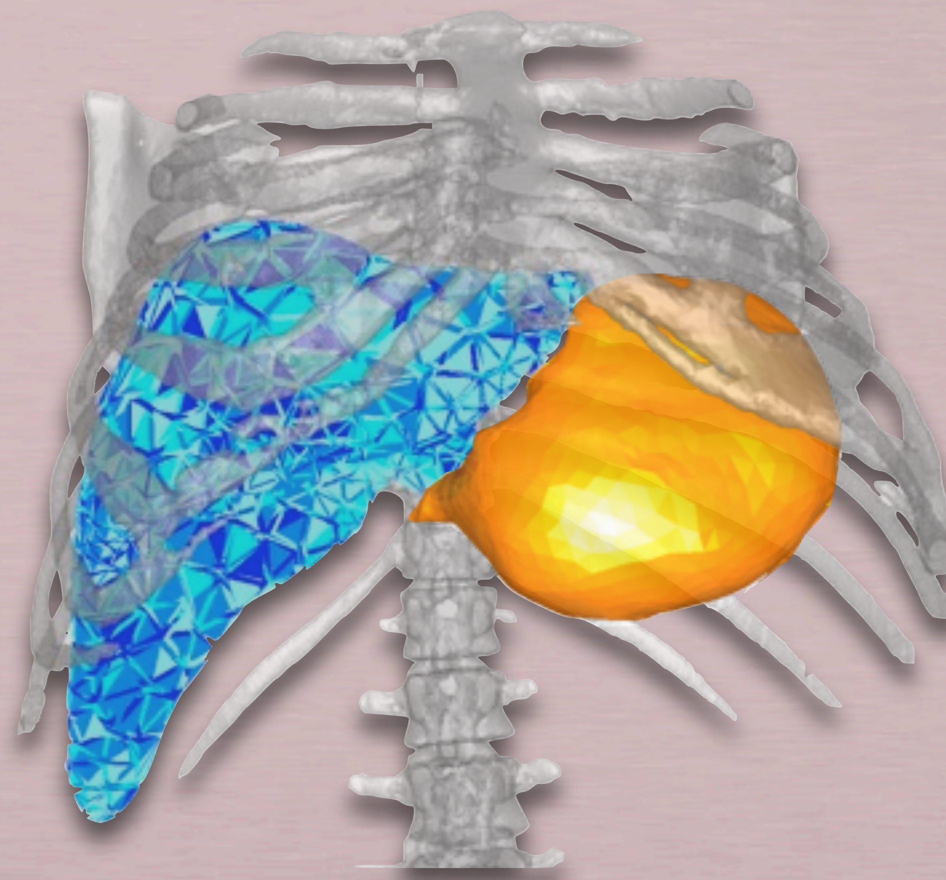




INTERACTION BETWEEN TWO DEFORMABLE MODELS

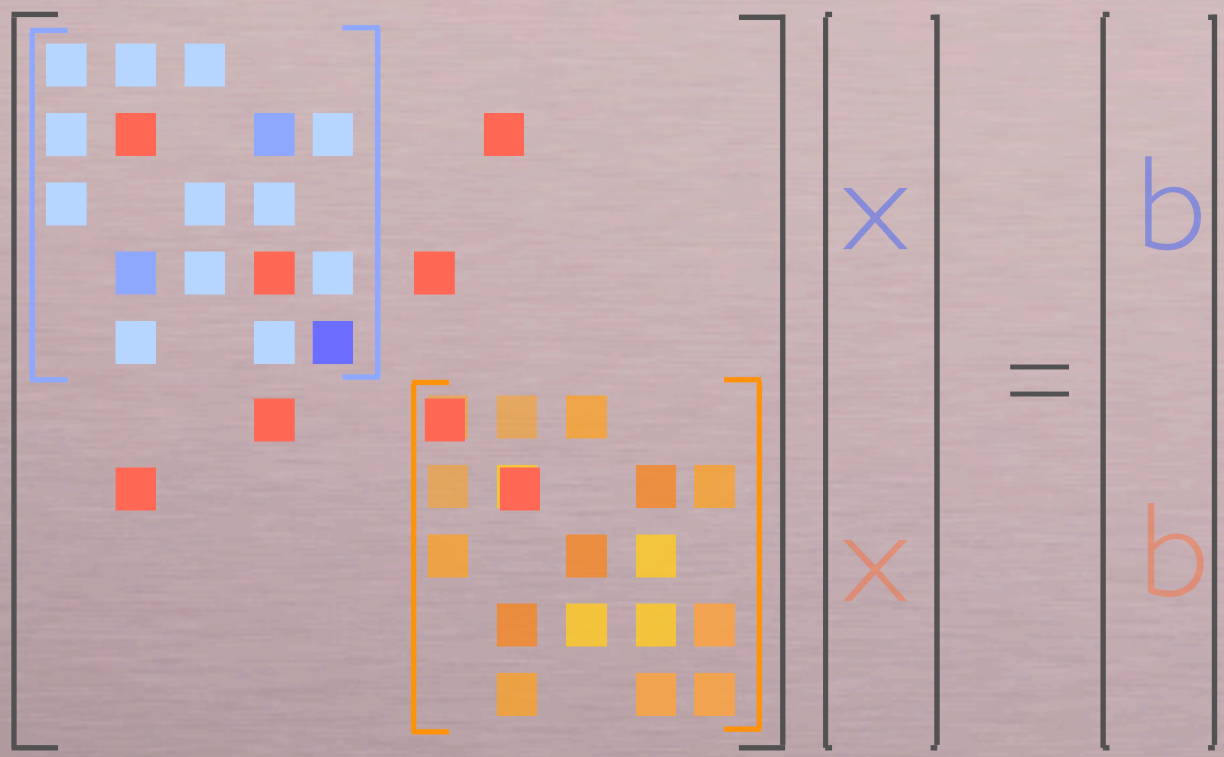
- 2 system of equations A x = b for each object
- «Direct approach»
  - Penalty forces: additional stiffness in the system
  - Lagrange Multipliers: very large system of (in)-equations

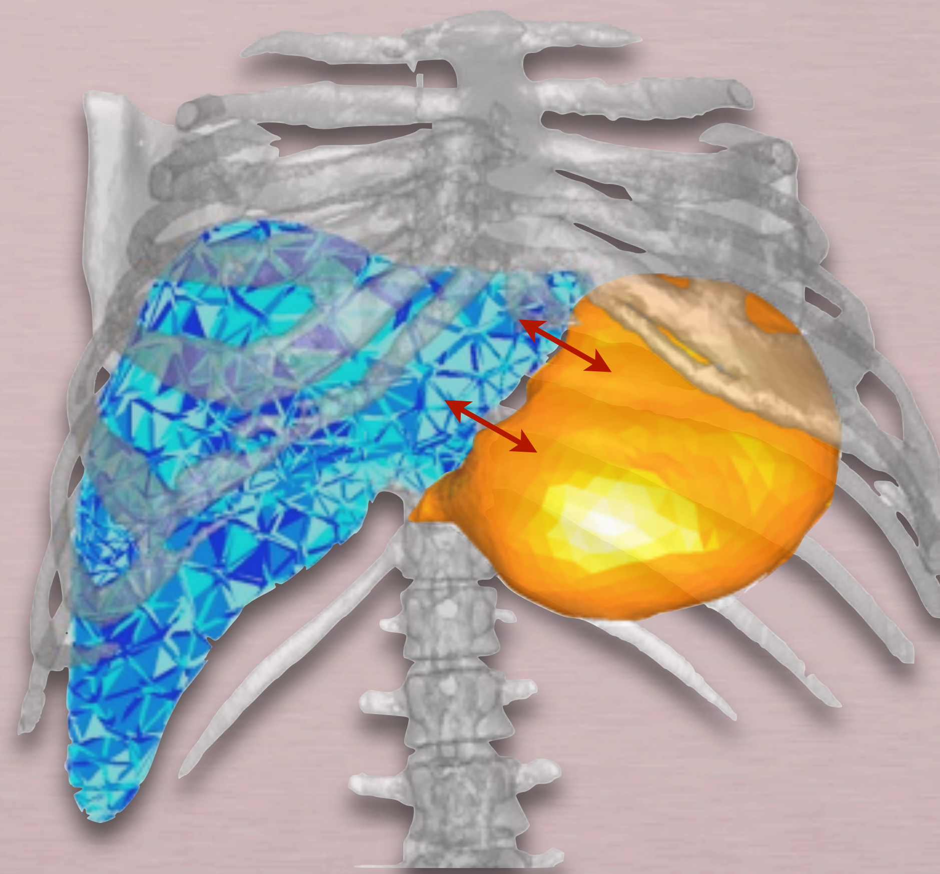




INTERACTION BETWEEN TWO DEFORMABLE MODELS

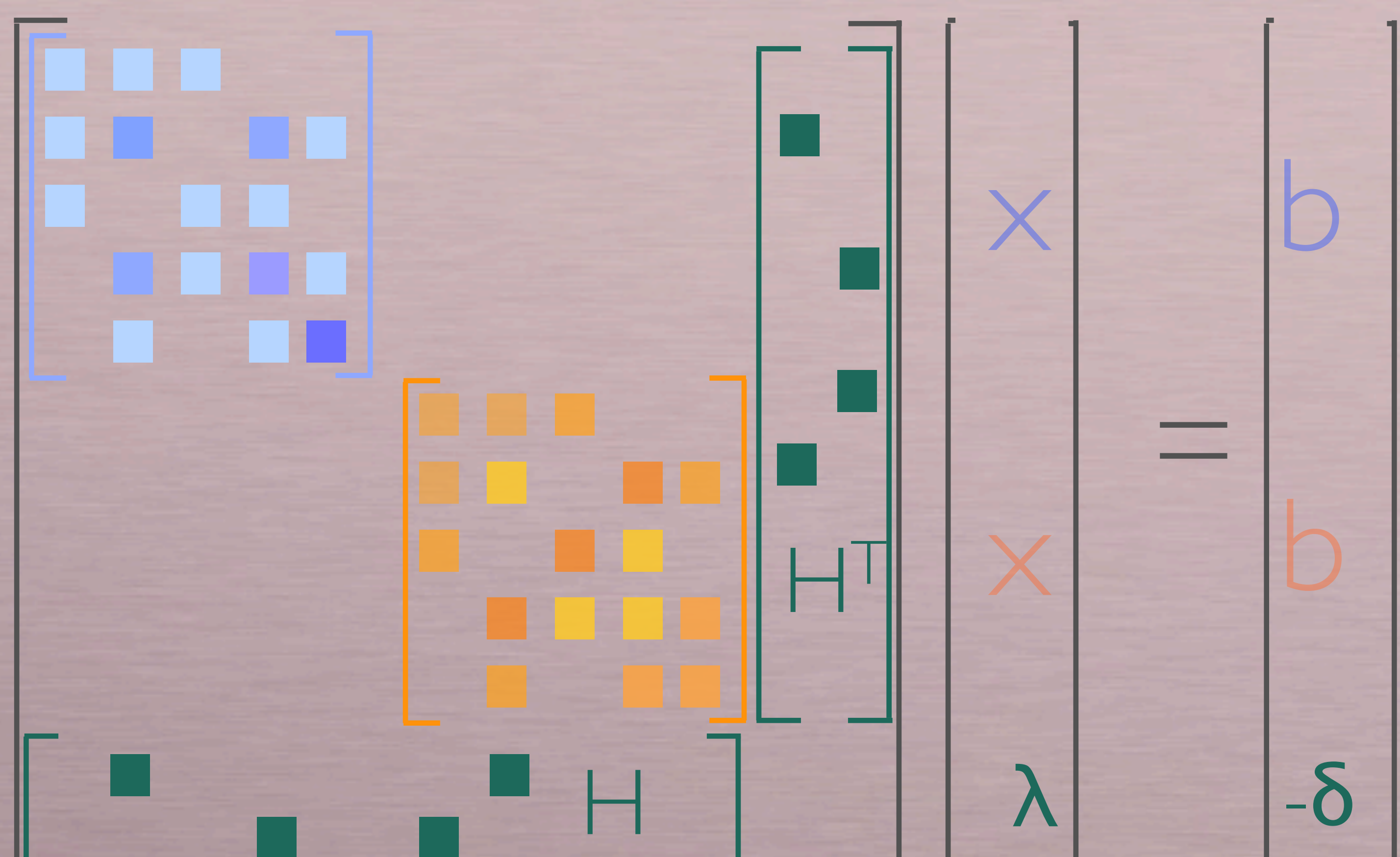
- 2 system of equations A x = b for each object
- «Direct approach»
  - Penalty forces: additional stiffness in the system
  - Lagrange Multipliers: very large system of (in)-equations

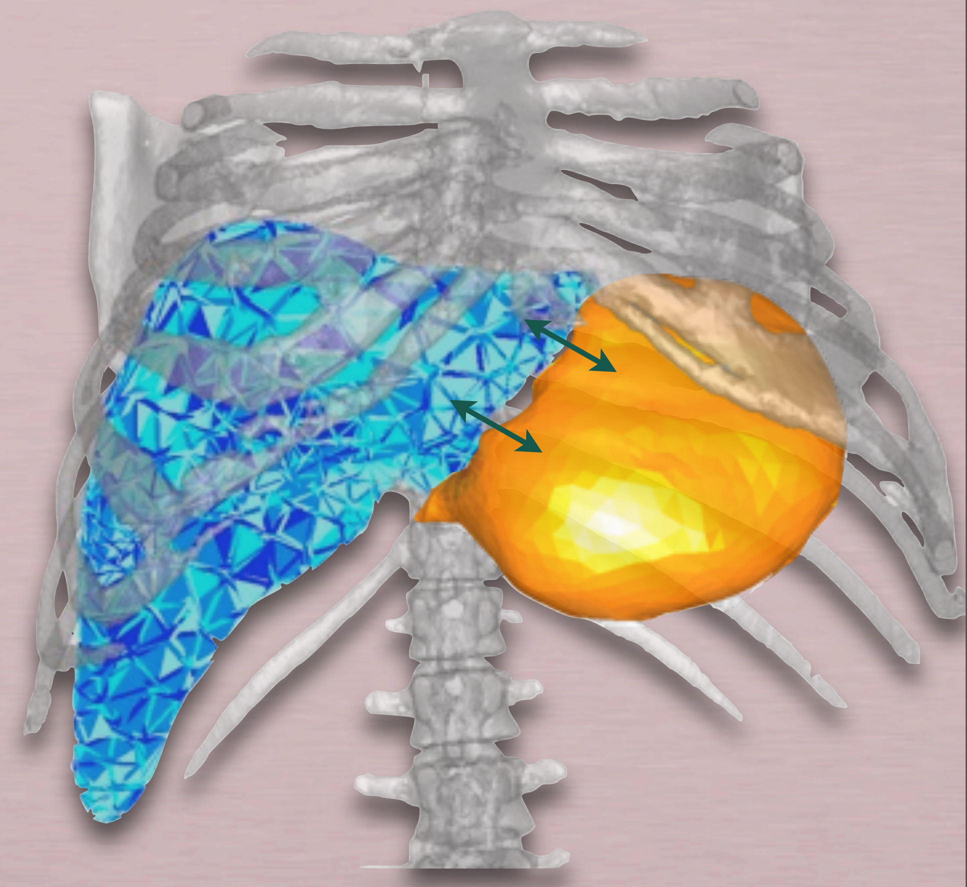




INTERACTION BETWEEN TWO DEFORMABLE MODELS

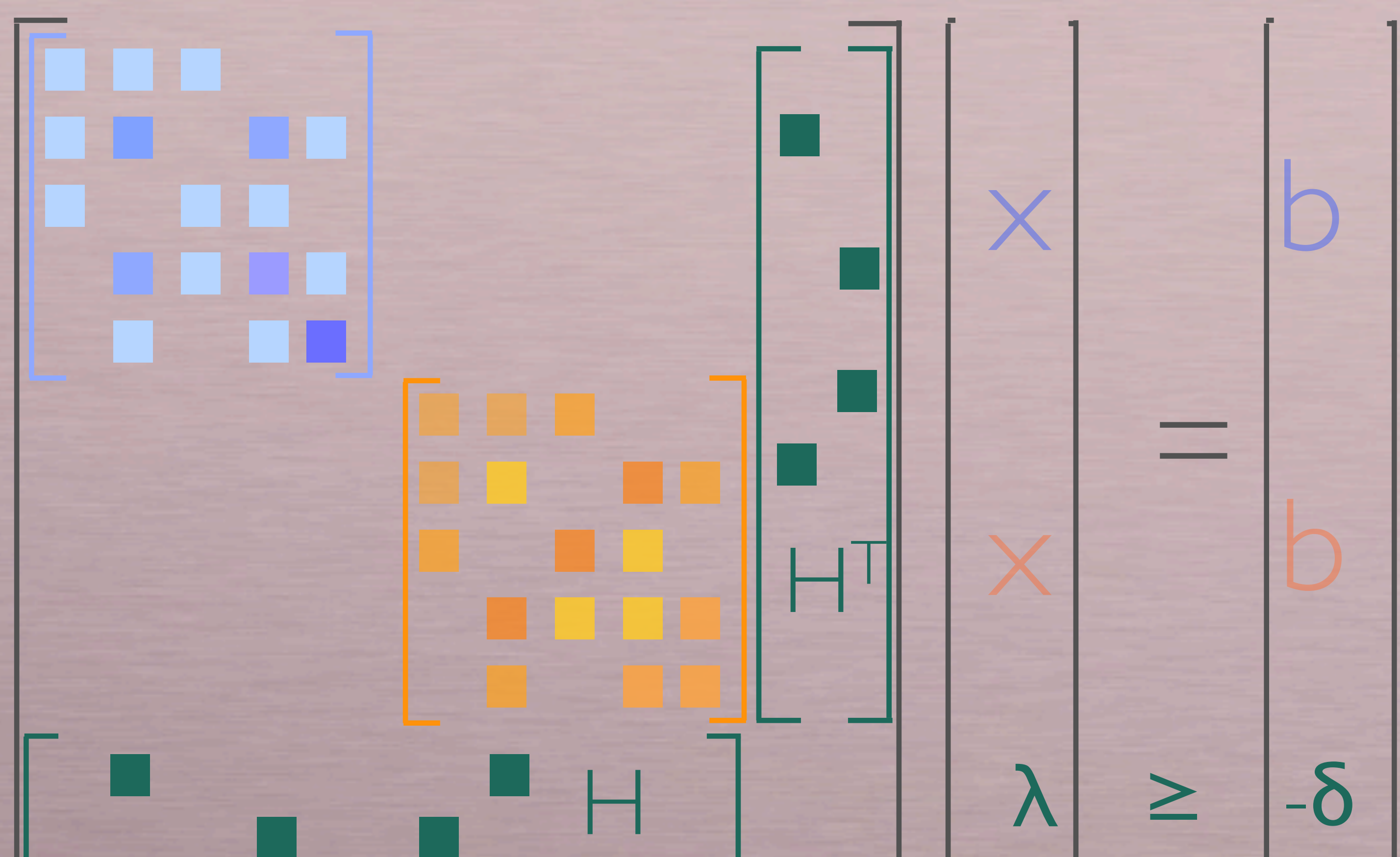
- 2 system of equations A x = b for each object
- «Direct approach»
  - Penalty forces: additional stiffness in the system
  - Lagrange Multipliers: very large system of (in)-equations

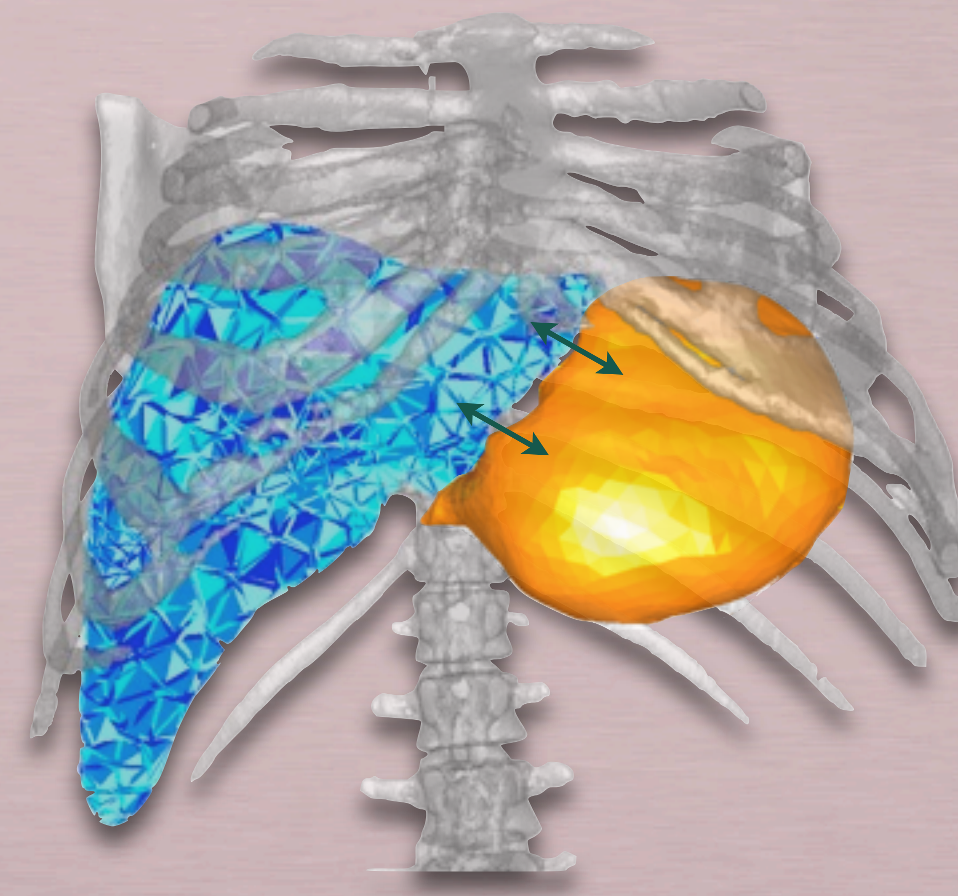




INTERACTION BETWEEN TWO DEFORMABLE MODELS

- 2 system of equations A x = b for each object
- «Direct approach»
  - Penalty forces: additional stiffness in the system
  - Lagrange Multipliers: very large system of (in)-equations





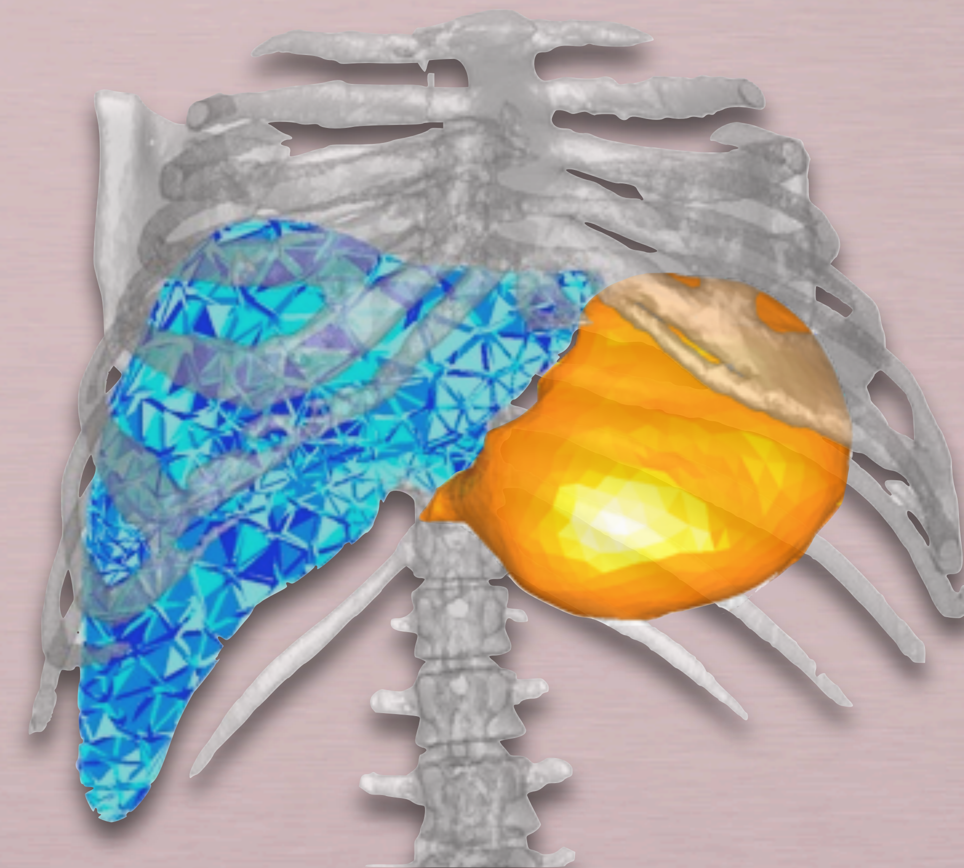
QP approach

INTERACTION BETWEEN TWO DEFORMABLE MODELS

- Indirect approach
  - Free Motion
  - Constraint setting
  - Constraint solving
  - Constraint correction: $x = x^{free} + Dx$

$$[A][x^{free}] = [b]$$

$$[A][x^{free}] = [b]$$

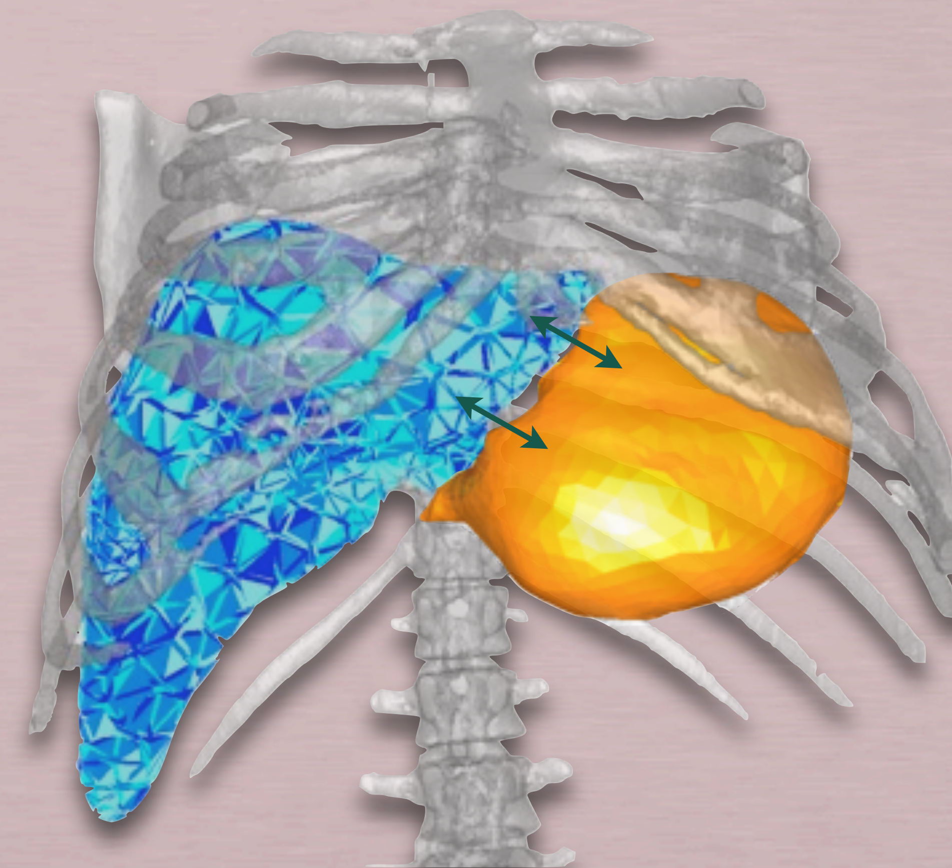


INTERACTION BETWEEN TWO DEFORMABLE MODELS

- Indirect approach
  - Free Motion
  - Constraint setting
  - Constraint solving
  - Constraint correction: $x = x^{free} + Dx$

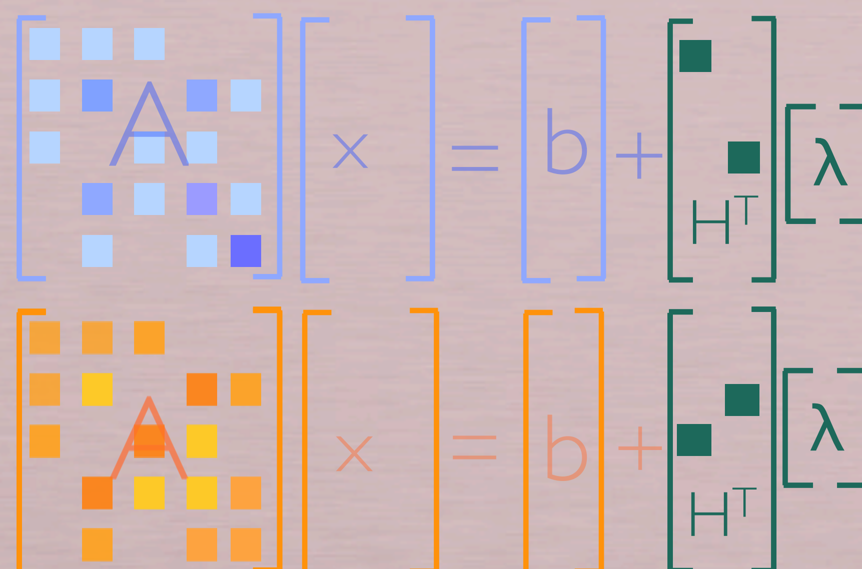
$$[A][x] = [b] + [H^T][\lambda]$$

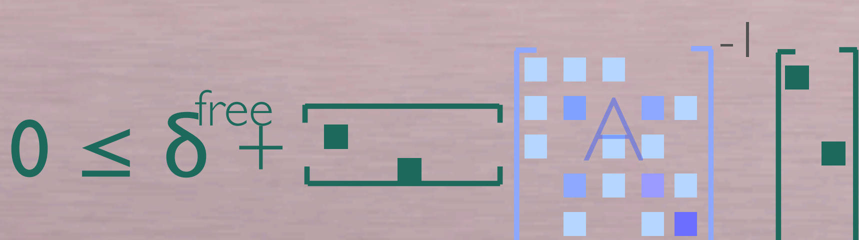
$$[A][x] = [b] + [H^T][\lambda]$$

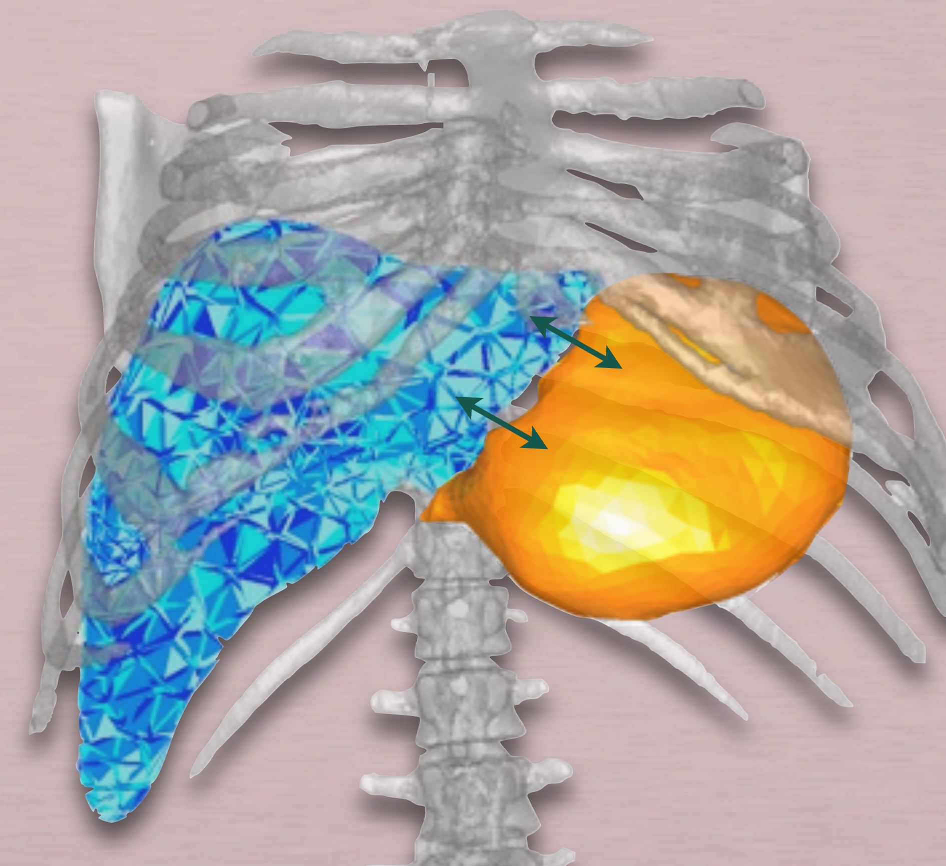


INTERACTION BETWEEN TWO DEFORMABLE MODELS

- Indirect approach
  - Free Motion
  - Constraint setting
  - Constraint solving
  - Constraint correction: $x = x^{free} + Dx$

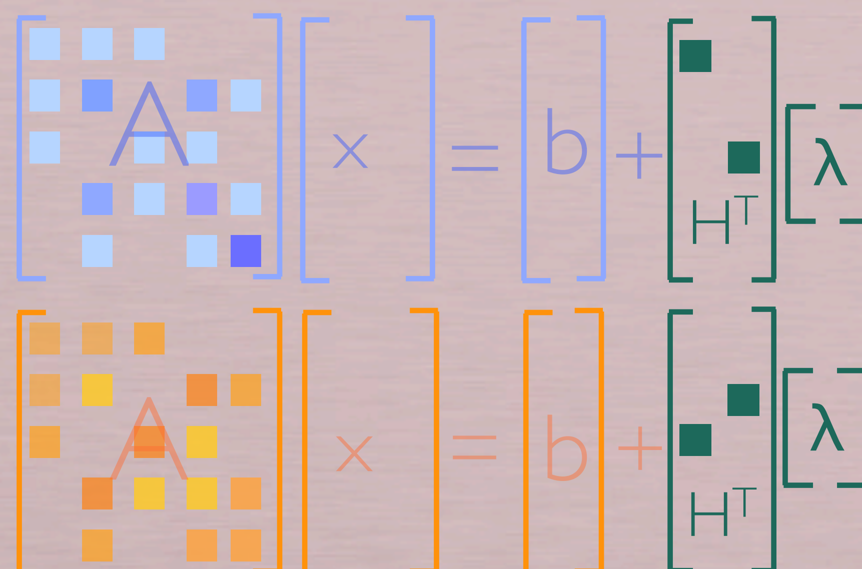


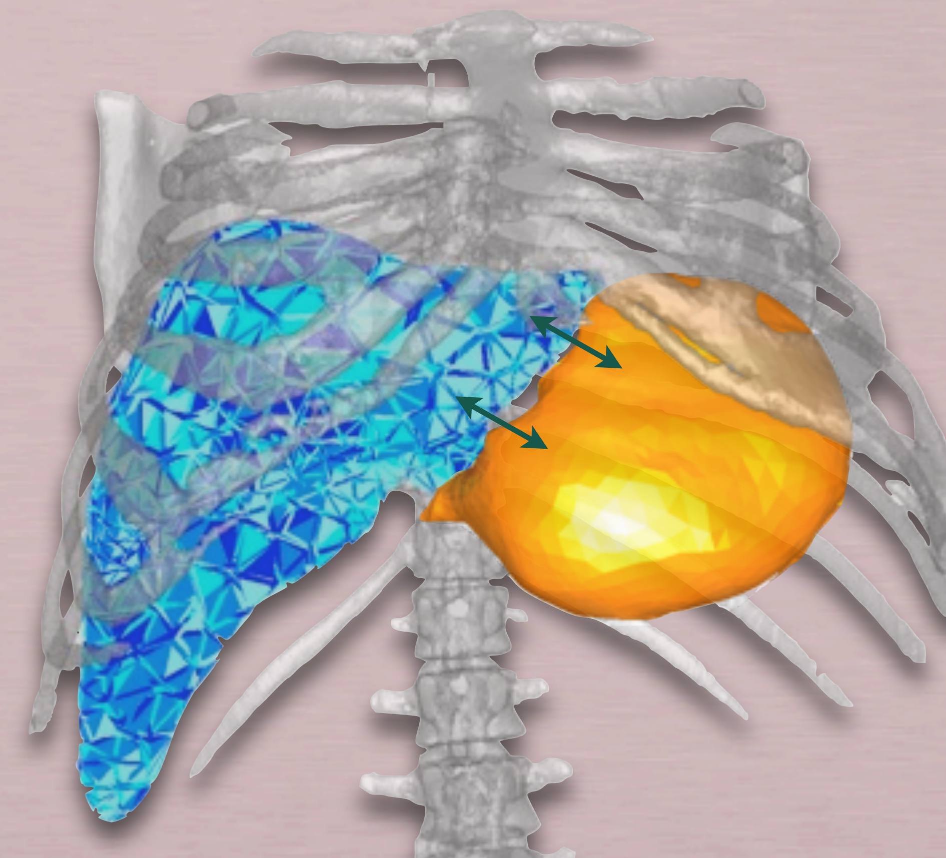


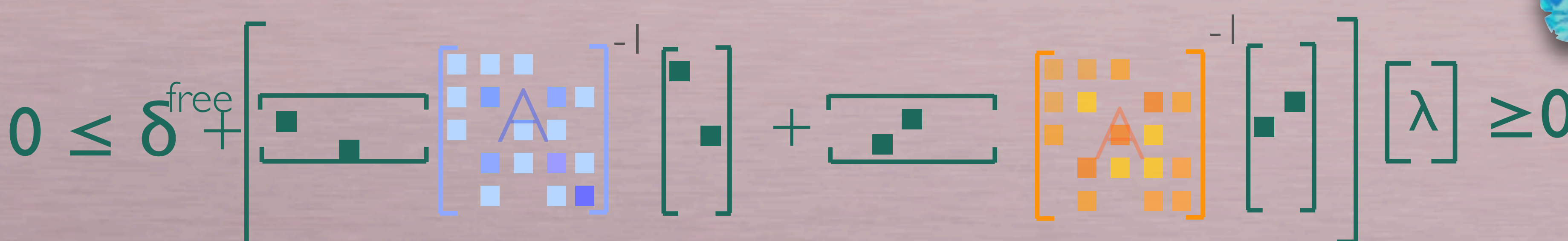


INTERACTION BETWEEN TWO DEFORMABLE MODELS

- Indirect approach
  - Free Motion
  - Constraint setting
  - Constraint solving
  - Constraint correction: $x = x^{free} + Dx$



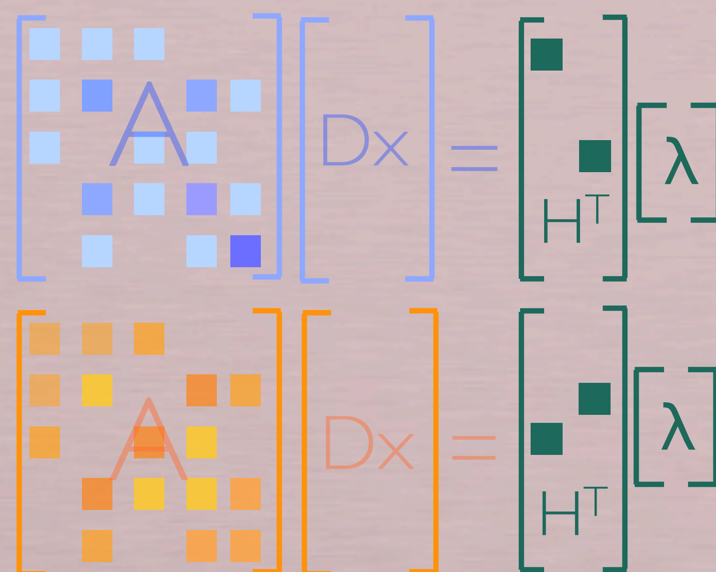


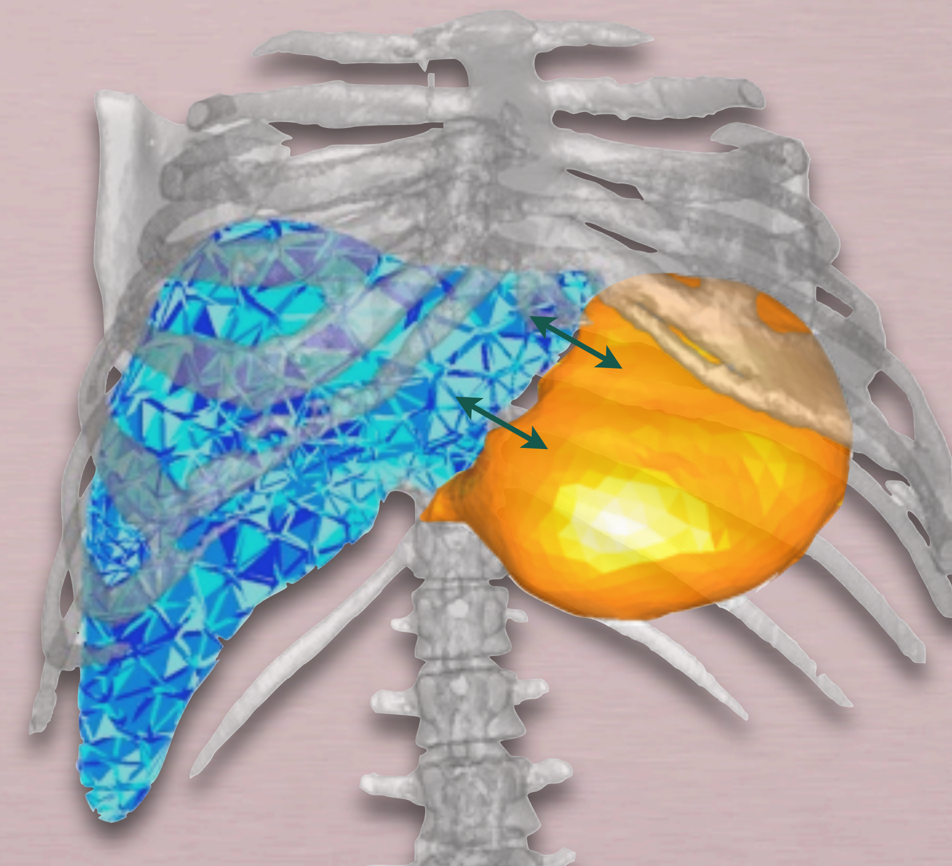


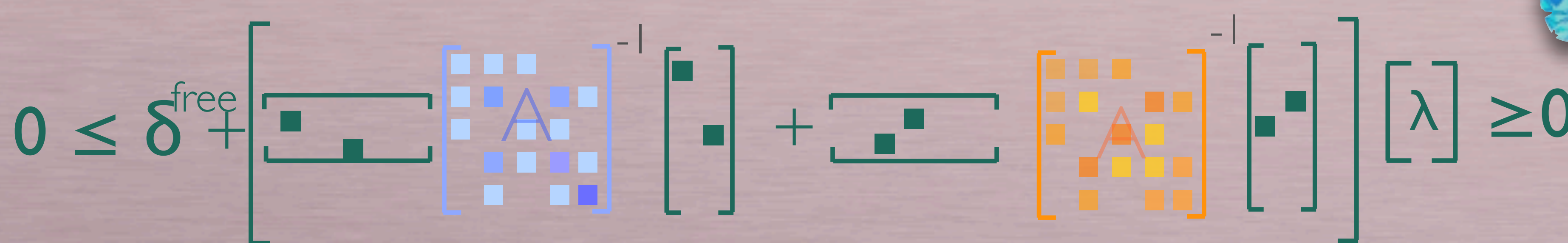
(N)LCP approach

INTERACTION BETWEEN TWO DEFORMABLE MODELS

- Indirect approach
  - Free Motion
  - Constraint setting
  - Constraint solving
  - Constraint correction: $x = x^{free} + Dx$







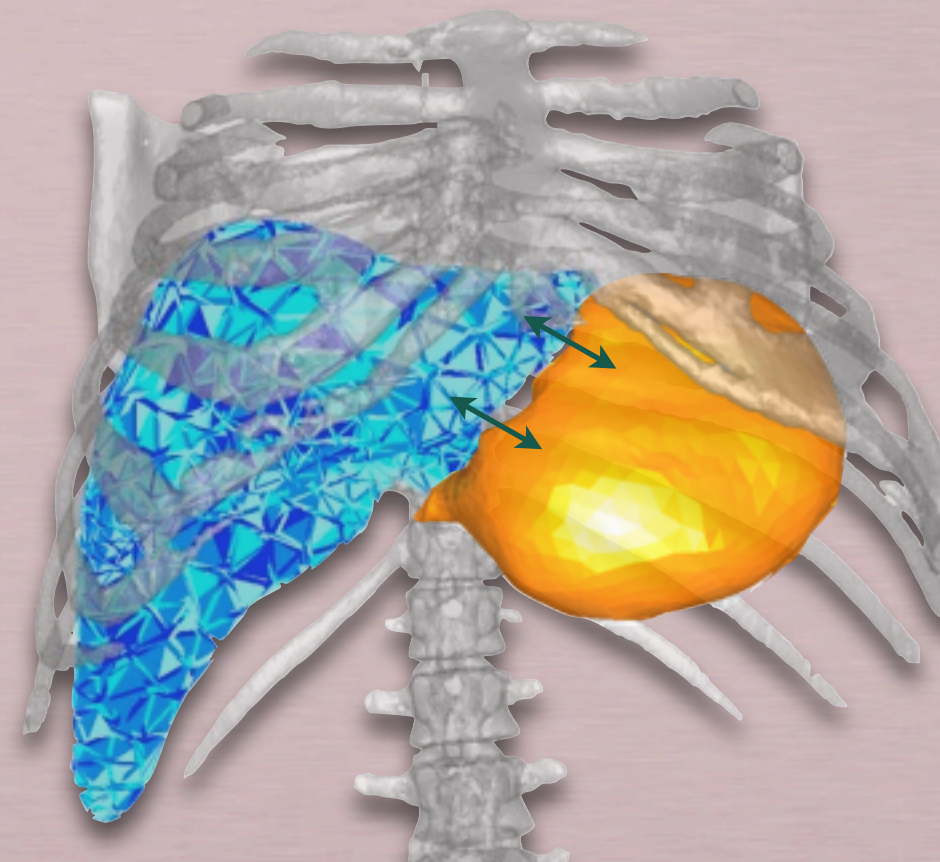
(N)LCP approach

INTERACTION BETWEEN TWO DEFORMABLE MODELS

- Indirect approach
  - Free Motion
  - Constraint setting
  - Constraint solving
  - Constraint correction: $x = x^{free} + Dx$

$$A \, Dx = H^T \lambda$$

$$A \, Dx = H^T \lambda$$



$$0 \leq \delta^{free} + \Big[\underbrace{H A^{-1} H^T}_{W} + \underbrace{H A^{-1} H^T}_{W} \Big] \lambda \geq 0$$

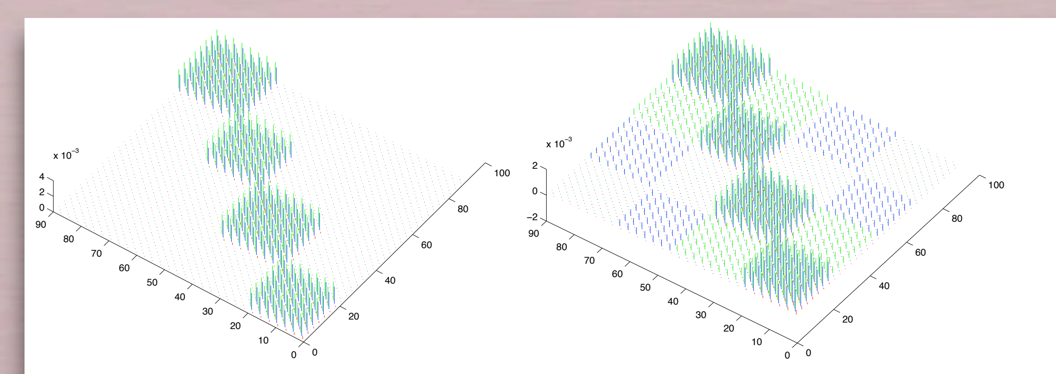
[Compliance in constraint space]

(N)LCP approach

(N)LCP APPROACH

- Build and solve (N)LCP
  - Direct solvers (Lemke) vs. iterative (Gauss-Seidel)
  - Inputs: W, δ^{free} (and constraints law), Output: λ

- What represents W ?
  - Mechanical coupling between constraints
  - Footstool example



- How to compute $W = H A^{-1} H^T$ for non-linear models ?
  - Linear model: A^{-1} can be precomputed
  - A is changing so computing A^{-1} in real-time is challenging !

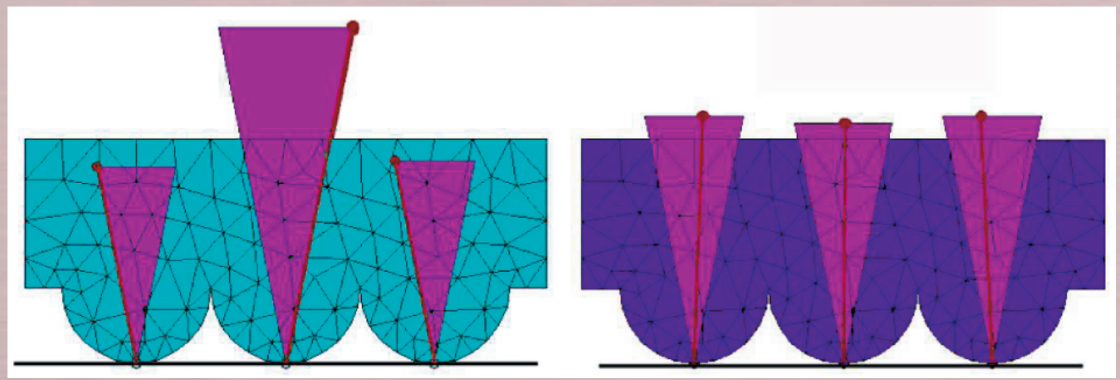
(N)LCP APPROACH

- Solve (N)LCP
  - Inputs: W, δ^{free} (and constraints law), Output: λ
  - Direct solvers
    - pivoting method: Lemke (see Siconos Library)
    - LCP <=> QP and use of QP solver algorithms
  - Iterative (block-Gauss-Seidel)
    - Block contact+friction
    - Iterative solver (slow convergence but it works well !)

$$\underbrace{\delta_\alpha - \mathbf{W}_{\alpha\alpha}\ \lambda_\alpha}_{\text{unknown}} = \underbrace{\sum_{\beta=1}^{\alpha-1} \mathbf{W}_{\alpha\beta}\ \lambda_\beta + \sum_{\beta=\alpha+1}^{m} \mathbf{W}_{\alpha\beta}\ \lambda_\beta}_{\text{frozen}} + \ \delta_\alpha^{\text{free}}$$

CHALLENGES...

- Non-unique solution (mainly with rigid objects)



- Non-linearity

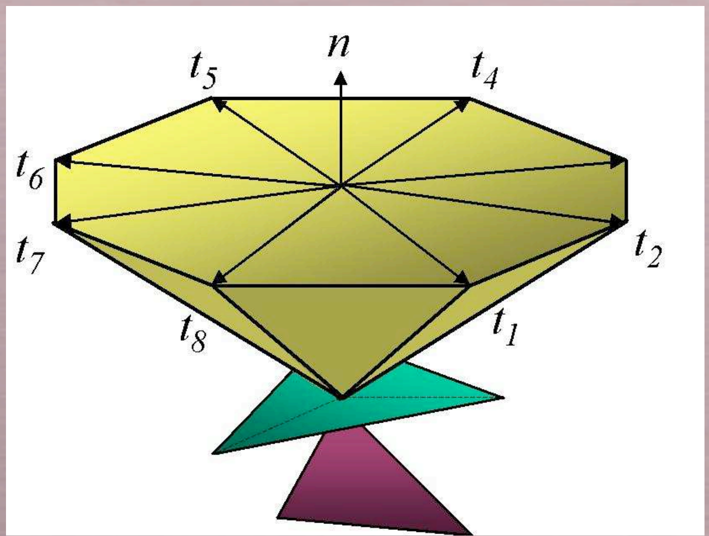
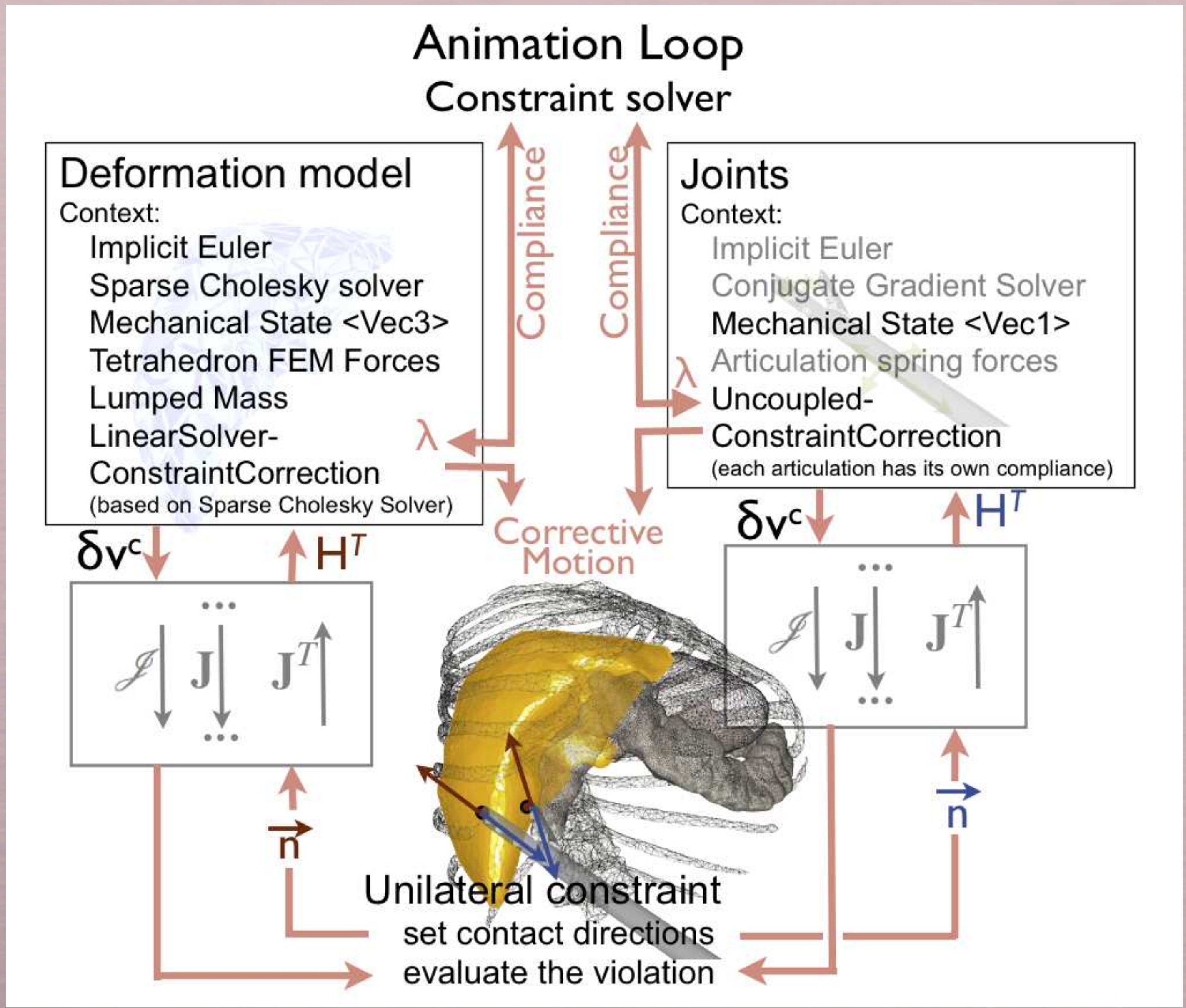


FIG. 2.27 – *Cône à $k = 8$ faces*

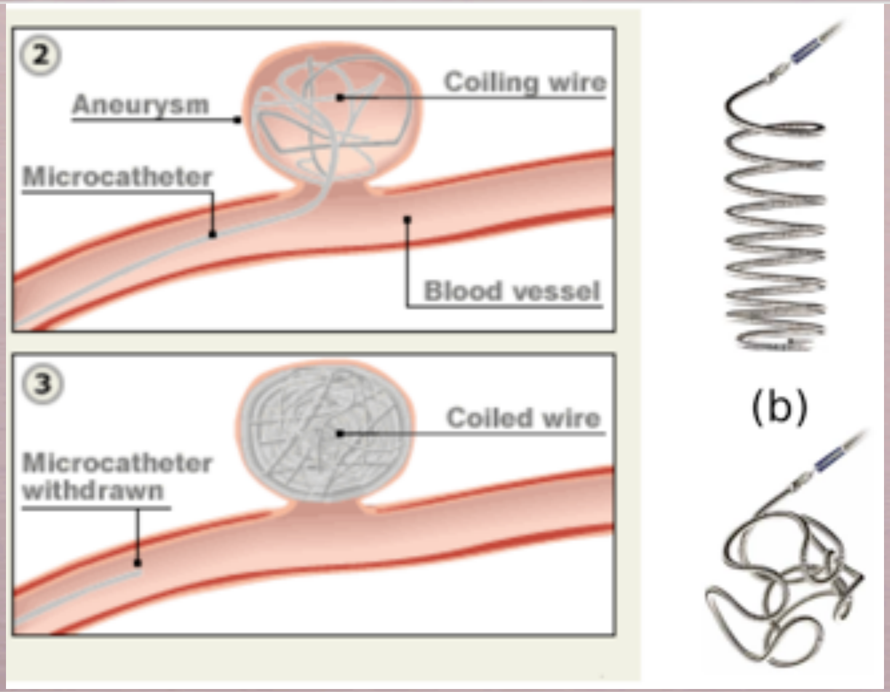
IMPLEMENTATION IN SOFA



COMPLIANCE FOR WIRE-LIKE STRUCTURES

- A is a block tri-diagonal matrix...
  - Order the contact along the curvilinear abscissa
  - Gauss-Seidel NLCP solver using unbuilt matrix $H\,A^{-1}H^{T}$:
  - Interactive simulation of more than 200 beams with hundreds of contacts
  - Frictional contact between coil and vessel wall

$$\delta_\alpha - \mathbf{W}_{\alpha\alpha}\lambda_\alpha = {}^{n}\mathbf{H}_\alpha^T \underbrace{\left[(\mathbf{A}^{-1})_{nn}\,\mathbf{r}_n + \sum_{i=1}^{n-1}(\mathbf{A}^{-1})_{ni}\,\mathbf{r}_i + \sum_{i=n+1}^{N-1}(\mathbf{A}^{-1})_{ni}\,\mathbf{r}_i\right]}_{\text{computed by substructure decomposition}} + \delta_\alpha^{\text{free}}$$

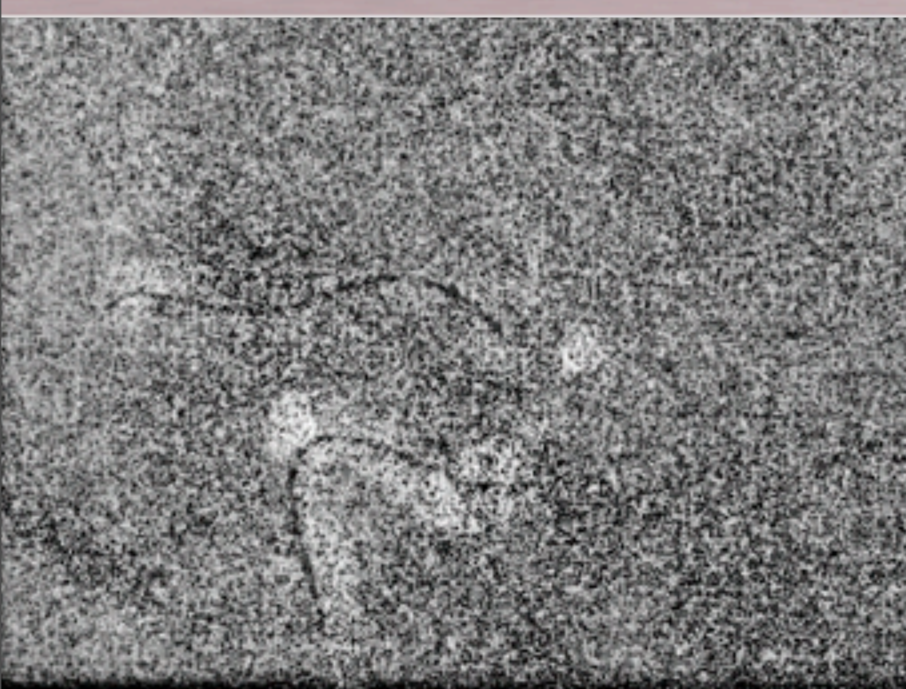


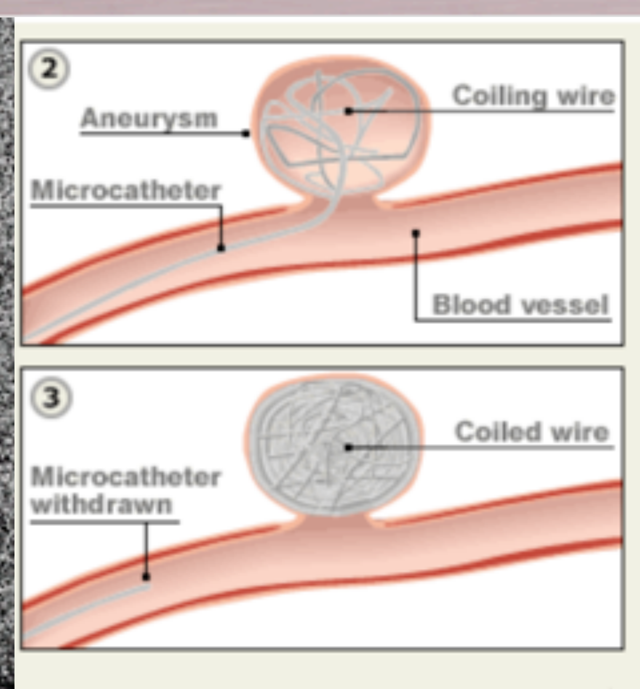
S. Cotin et al. MICCAI 2005
C. *Duriez et al.* Computer Aided Surgery Journal 2006
J. *Dequidt et al.* MICCAI 2008 & MICCAI 2009

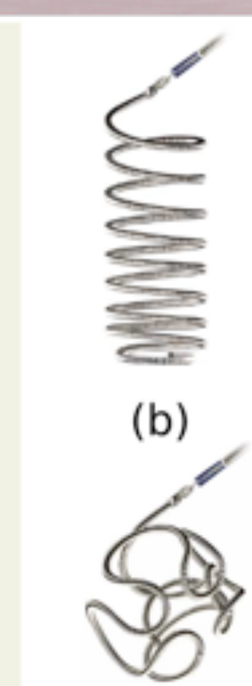
COMPLIANCE FOR WIRE-LIKE STRUCTURES

- A is a block tri-diagonal matrix...
  - Order the contact along the curvilinear abscissa
  - Gauss-Seidel NLCP solver using unbuilt matrix $H\,A^{-1}H^{T}$:
  - Interactive simulation of more than 200 beams with hundreds of contacts
  - Frictional contact between coil and vessel wall

$$\delta_\alpha - \mathbf{W}_{\alpha\alpha}\lambda_\alpha = {}^{n}\mathbf{H}_\alpha^T \underbrace{\left[(\mathbf{A}^{-1})_{nn}\,\mathbf{r}_n + \sum_{i=1}^{n-1} (\mathbf{A}^{-1})_{ni}\,\mathbf{r}_i + \sum_{i=n+1}^{N-1} (\mathbf{A}^{-1})_{ni}\,\mathbf{r}_i \right]}_{\text{computed by substructure decomposition}} + \delta_\alpha^{\text{free}}$$

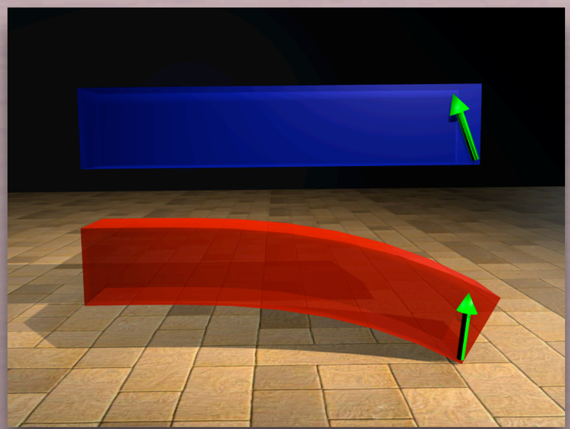






COMPLIANCE WARPING

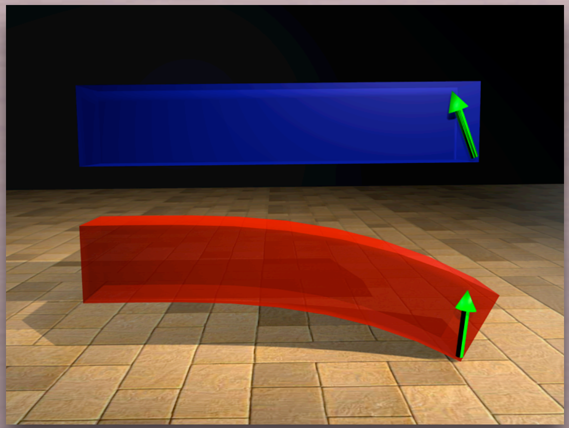
- Computation of $W=H\,A^{-1}H^T$ on a volume deformable object ?
  - In general, too long for real-time
  - The main role of W is to get the mechanical coupling between contacts.
- Could we «precompute» A^{-1}?
  - No... as we are using non-linear model, A is changing
  - Yes... but it is the approximation that the compliance is only «rotated» by deformation: $A^{-1} \sim R\,A_0^{-1}Rt$

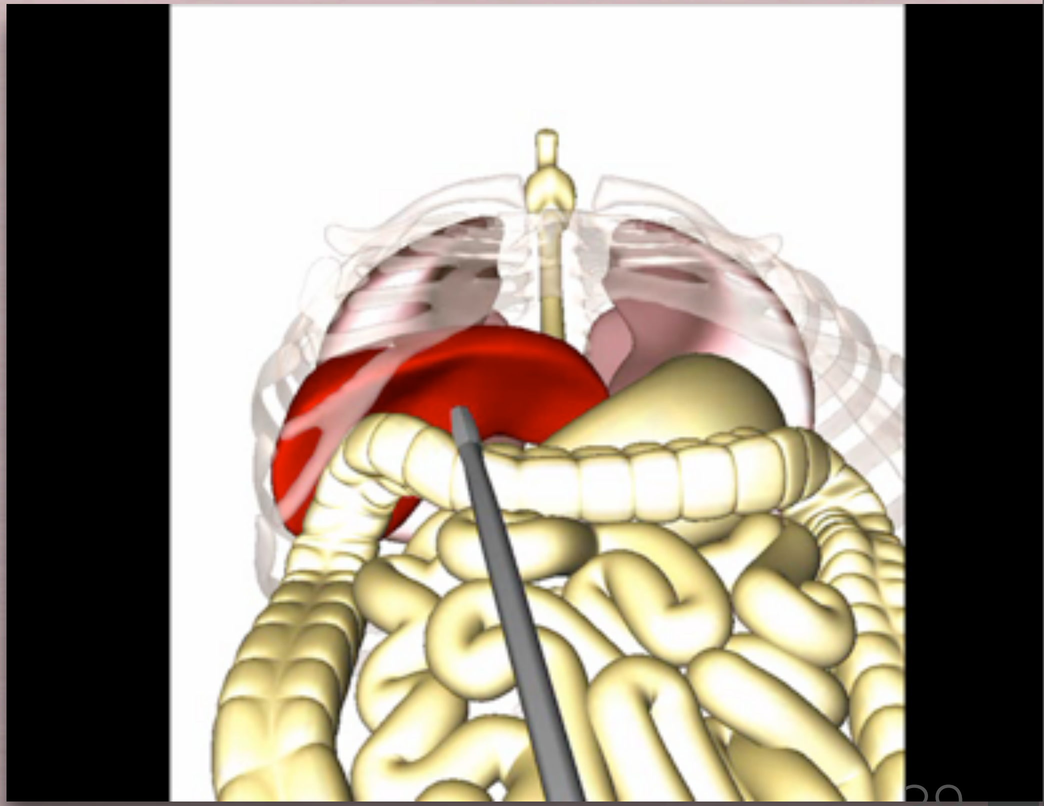


Saupin et al. CGI 2008

COMPLIANCE WARPING

- Computation of $W = H A^{-1} H^T$ on a volume deformable object ?
  - In general, too long for real-time
  - The main role of W is to get the mechanical coupling between contacts.
- Could we «precompute» A^{-1}?
  - No... as we are using non-linear model, A is changing
  - Yes... but it is the approximation that the compliance is only «rotated» by deformation: $A^{-1} \sim R A_0^{-1} Rt$





Saupin et al. CGI 2008

COMPLIANCE BASED ON ASYNCHRONOUS PRECONDITIONER

- The compliance matrix can be approximated by preconditionning technique:

$$\mathbf{W} \approx h^2 \, \mathbf{H} \, (\mathbf{LDL})^{-1} \, \mathbf{H}^T$$

- Use of the (asynchronous) preconditioner technique

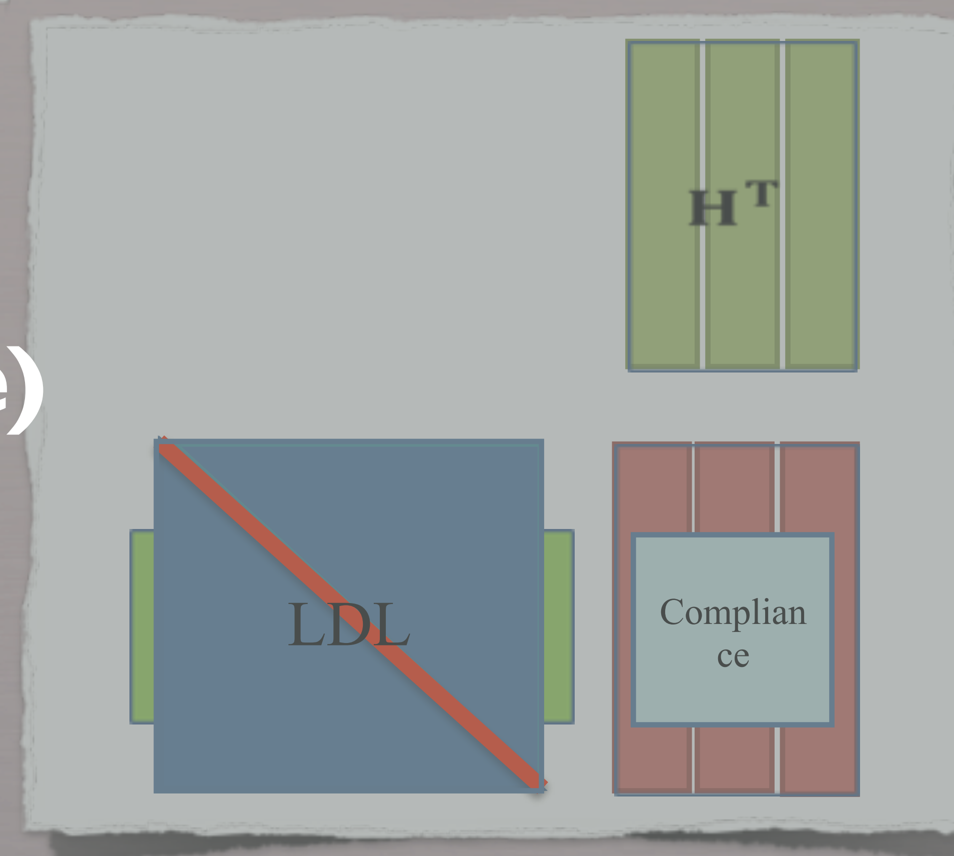
  1. Repeat this operation, until get $S = (LDL)^{-1}H^T$

  2. Finally, obtain the compliance matrix with:

$$W = H\,(LDL)^{-1}H^T = H\,S$$

GPU....

(tout à l'heure)



Courtecuisse et al. MICCAI 2011

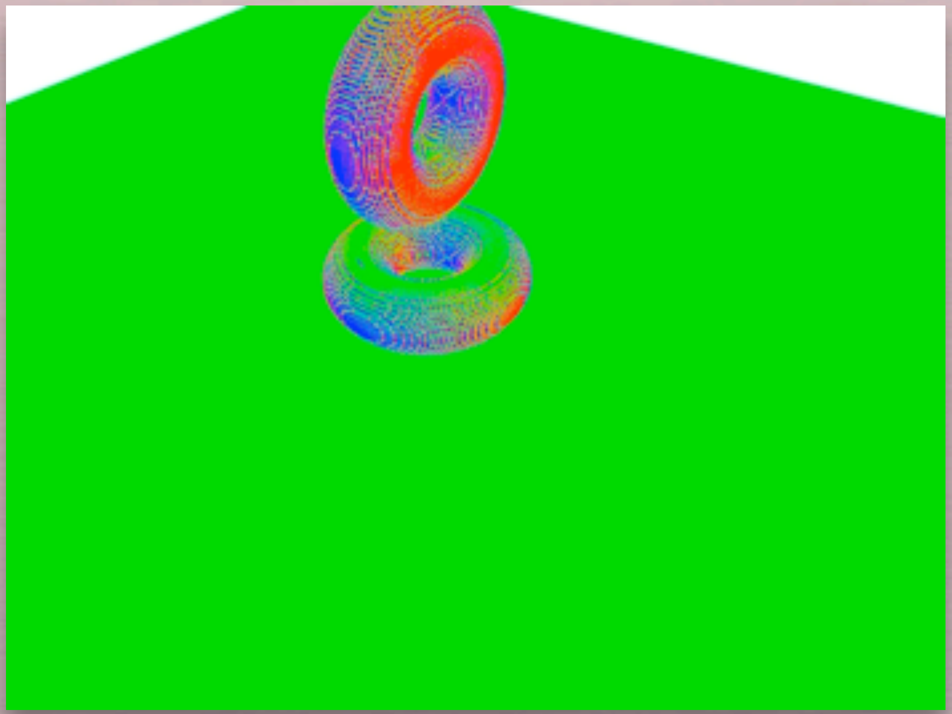
VOLUME CONTACT MODEL AT ARBITRARY RESOLUTION

- **Goal: reduce the size of W using less contact constraints**
- Volume of Interpenetration instead of distance
- Algorithm
  - Use GPU to render the simulation scene along 3 directions
  - This generates a volumetric image
  - From this image we identify the intersection volumes
  - These volumes are used to compute the collision response (forces)
- Independent from the mesh resolution
- No additional cost to detect self-collisions

Allard et al. SIGGRAPH 2010

VOLUME CONTACT MODEL AT ARBITRARY RESOLUTION

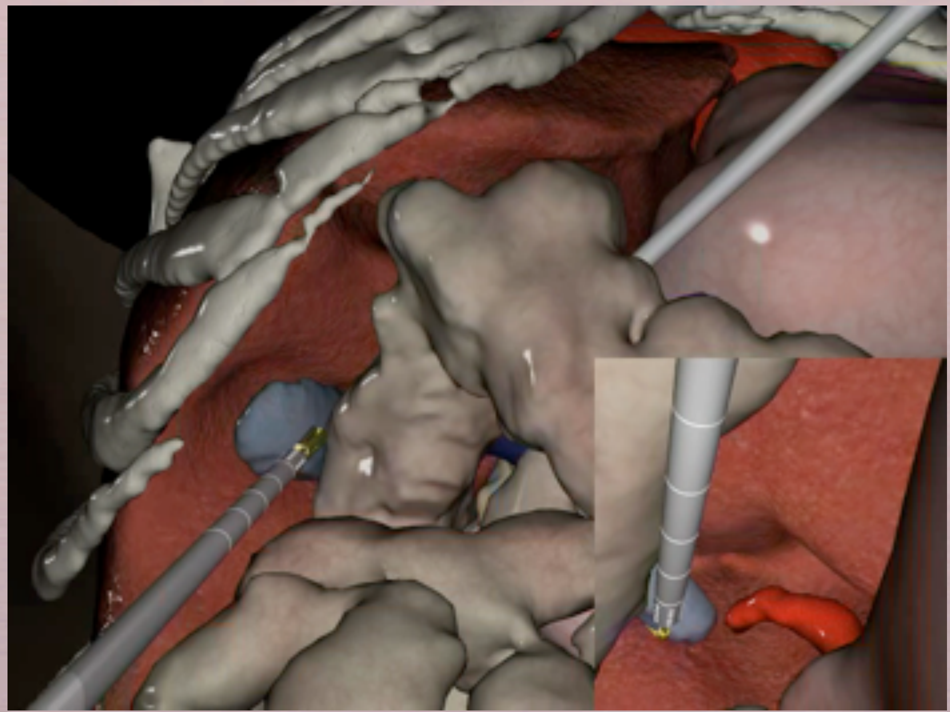
- **Goal: reduce the size of W using less contact constraints**
- Volume of Interpenetration instead of distance
- Algorithm
  - Use GPU to render the simulation scene along 3 directions
  - This generates a volumetric image
  - From this image we identify the intersection volumes
  - These volumes are used to compute the collision response (forces)
- Independent from the mesh resolution
- No additional cost to detect self-collisions



Allard et al. SIGGRAPH 2010

VOLUME CONTACT MODEL AT ARBITRARY RESOLUTION

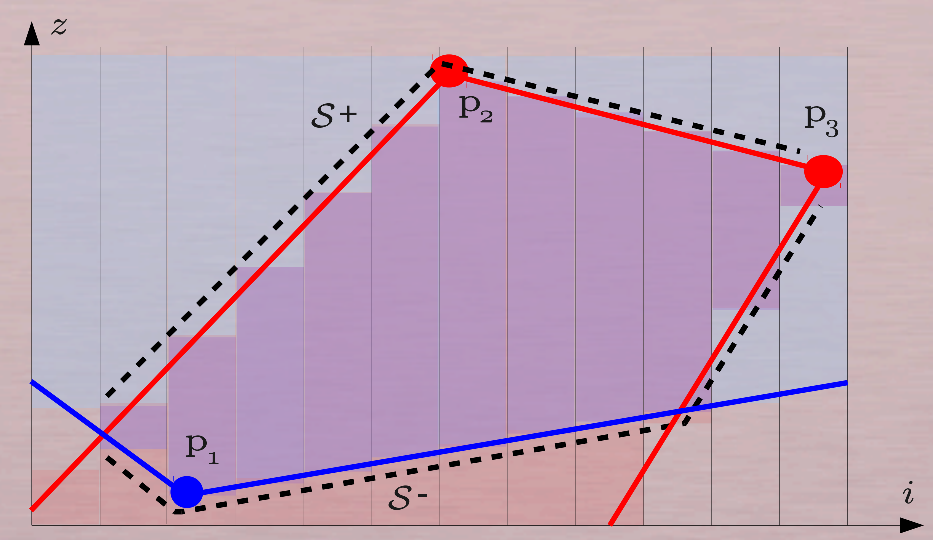
- **Goal: reduce the size of W using less contact constraints**
- Volume of Interpenetration instead of distance
- Algorithm
  - Use GPU to render the simulation scene along 3 directions
  - This generates a volumetric image
  - From this image we identify the intersection volumes
  - These volumes are used to compute the collision response (forces)
- Independent from the mesh resolution
- No additional cost to detect self-collisions



Allard et al. SIGGRAPH 2010

VOLUME CONTACT MODEL AT ARBITRARY RESOLUTION

- **Goal: reduce the size of W using less contact constraints**
- Volume of Interpenetration instead of distance
- Algorithm
  - Use GPU to render the simulation scene along 3 directions
  - This generates a volumetric image
  - From this image we identify the intersection volumes
  - These volumes are used to compute the collision response (forces)
- Independent from the mesh resolution
- No additional cost to detect self-collisions



Allard et al. SIGGRAPH 2010

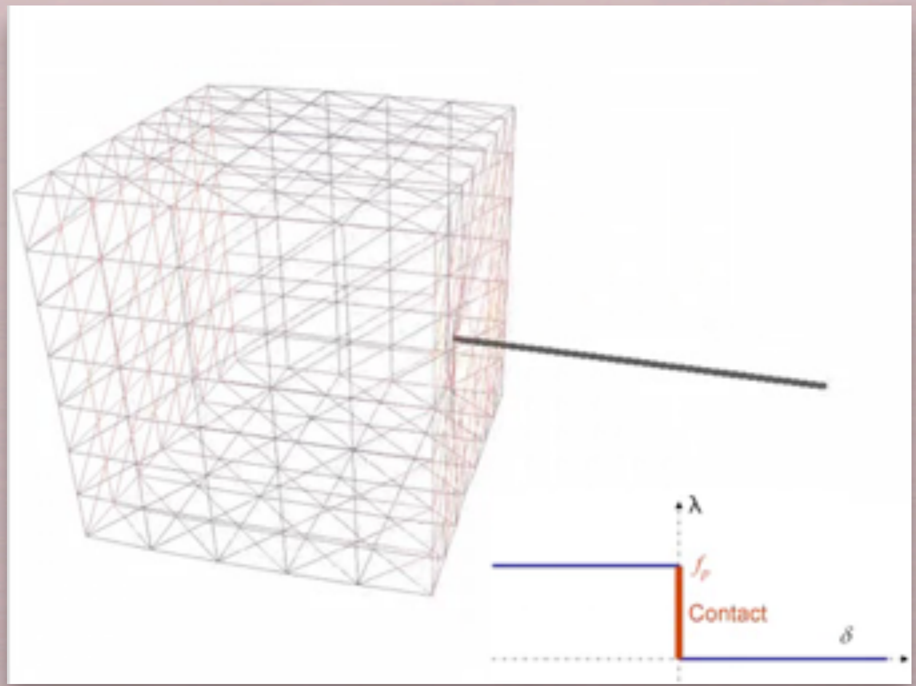
CONSTRAINT-BASED NEEDLE-INSERTION MODEL

- Many surgical techniques involve needle insertion (biopsy, brachytherapy, ...)
  - These needles are rigid or flexible
  - Different constraint laws for puncture, cutting, friction... etc...
  - Results validated by comparing with experiments of the literature
- Suturing simulation
  - The beam model can be used for the whole suture (including the needle)
  - Constraint based approach to simulate the suturing
  - Work in progress with DigitalTrainers
- NLCP Generic solver for all type of mechanical interactions

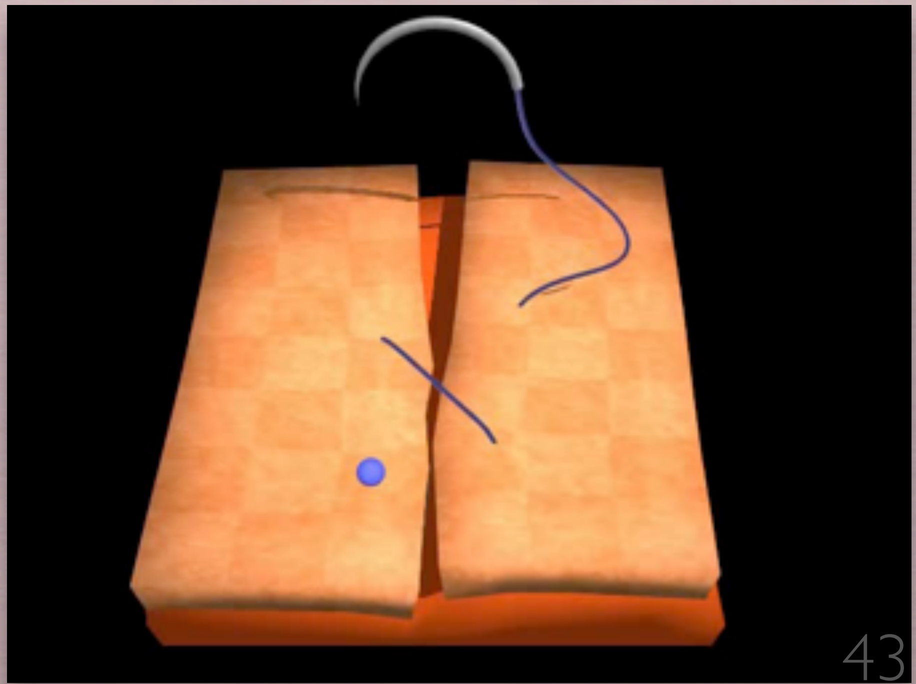
Guébert et al Vriphys 2008
Duriez et al. MICCAI 2009
Guébert et al SCA 2009

CONSTRAINT-BASED NEEDLE-INSERTION MODEL

- Many surgical techniques involve needle insertion (biopsy, brachytherapy, ...)
  - These needles are rigid or flexible
  - Different constraint laws for puncture, cutting, friction... etc...
  - Results validated by comparing with experiments of the literature



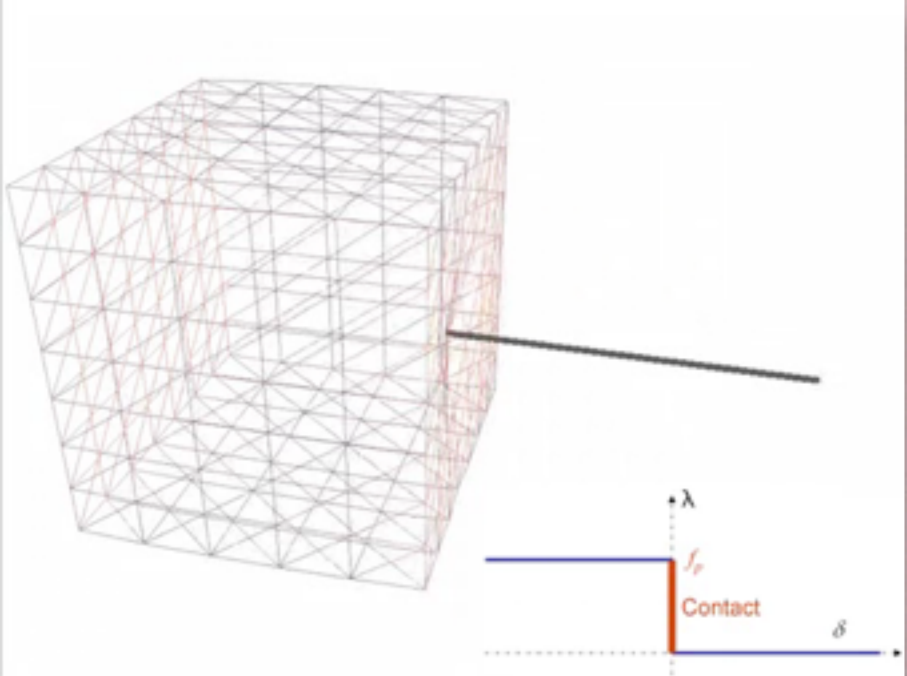
- Suturing simulation
  - The beam model can be used for the whole suture (including the needle)
  - Constraint based approach to simulate the suturing
  - Work in progress with DigitalTrainers
- NLCP Generic solver for all type of mechanical interactions



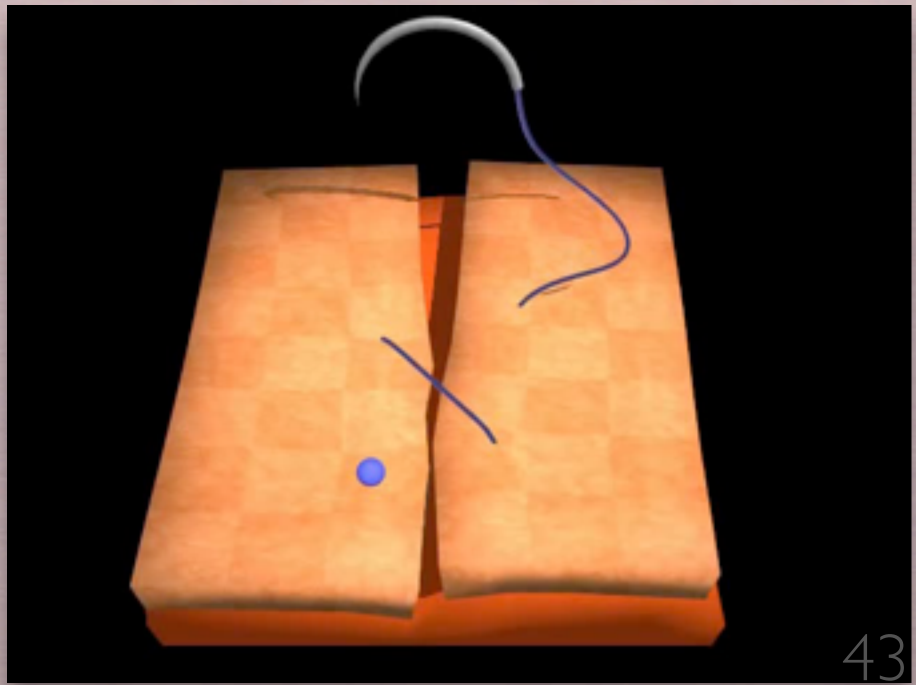
Guébert et al Vriphys 2008
Duriez et al. MICCAI 2009
Guébert et al SCA 2009

CONSTRAINT-BASED NEEDLE-INSERTION MODEL

- Many surgical techniques involve needle insertion (biopsy, brachytherapy, ...)
  - These needles are rigid or flexible
  - Different constraint laws for puncture, cutting, friction... etc...
  - Results validated by comparing with experiments of the literature



- Suturing simulation
  - The beam model can be used for the whole suture (including the needle)
  - Constraint based approach to simulate the suturing
  - Work in progress with DigitalTrainers

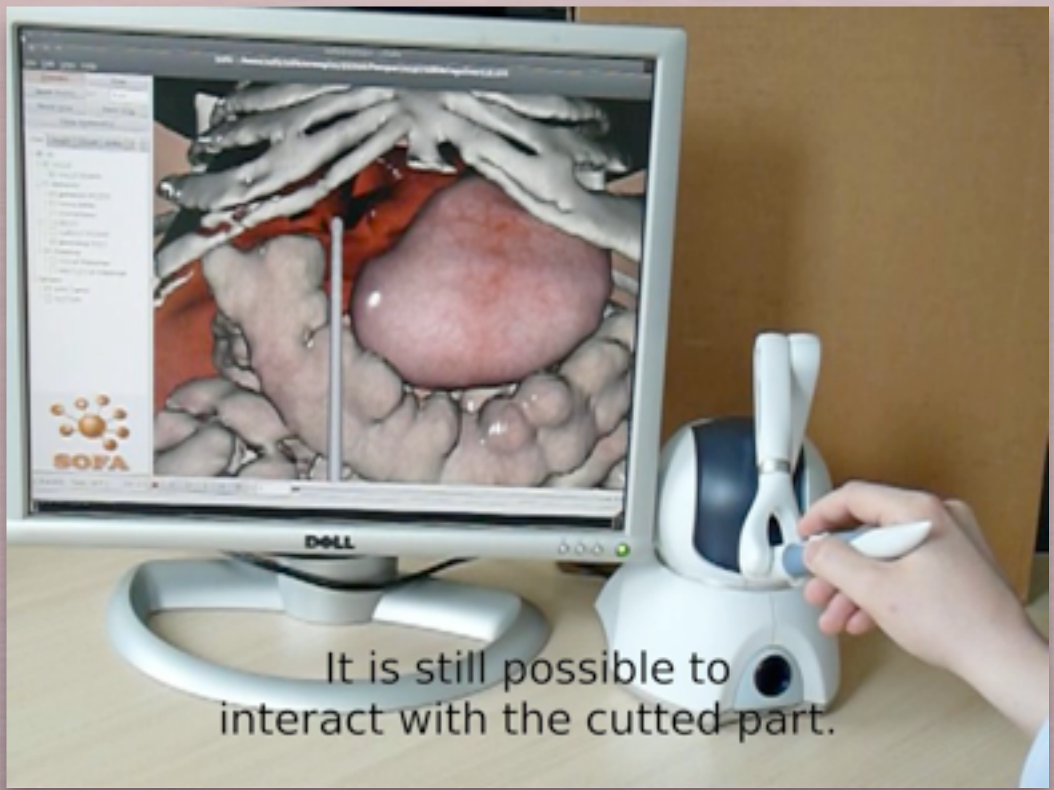


- NLCP Generic solver for all type of mechanical interactions

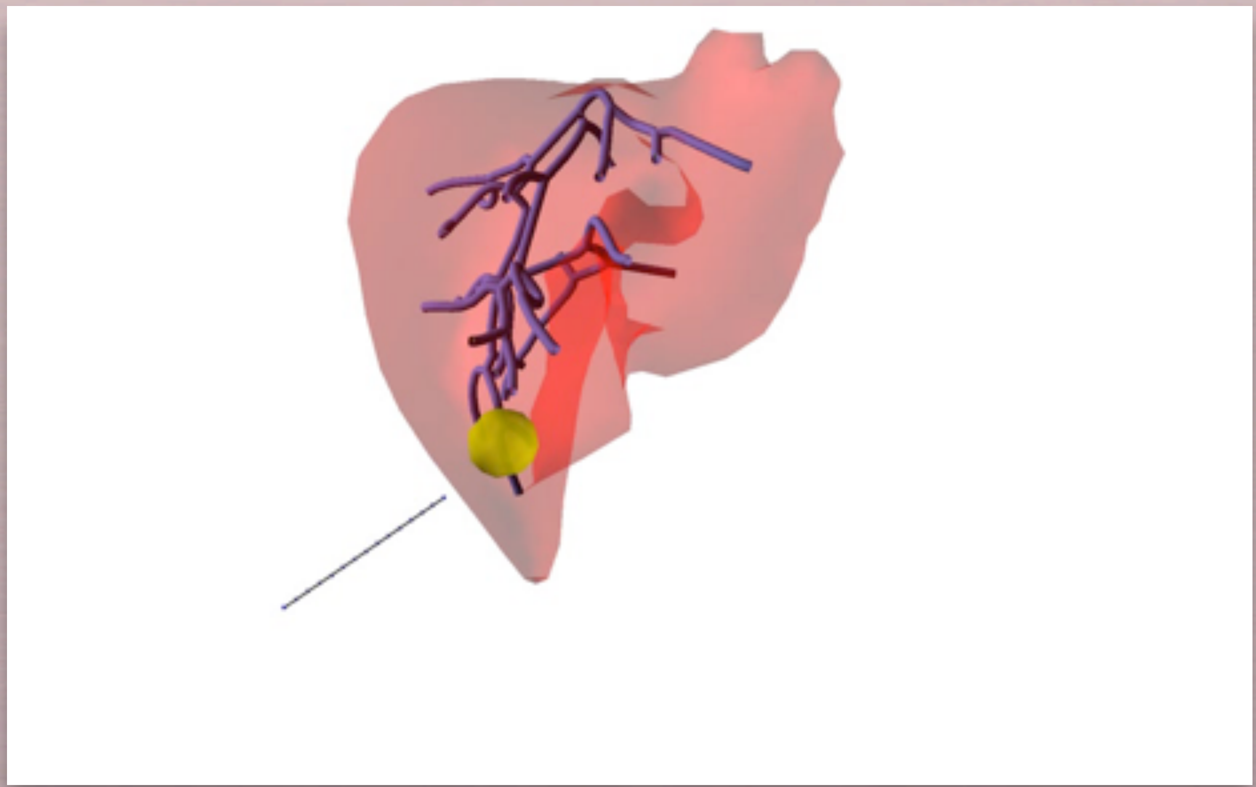
Guébert et al Vriphys 2008
Duriez et al. MICCAI 2009
Guébert et al SCA 2009

LIVER SIMULATION

All these method can be combined...



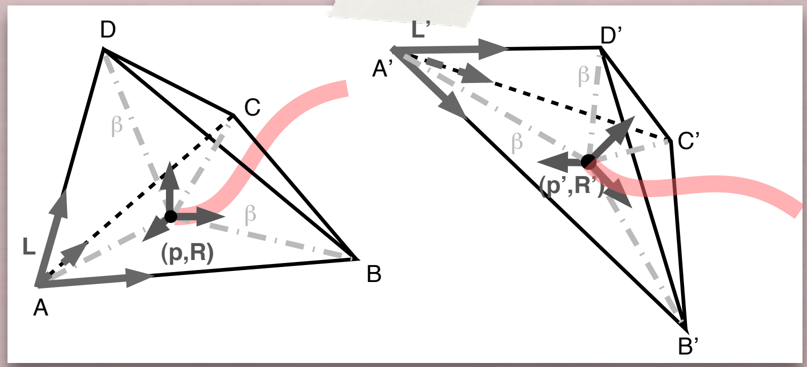
Asynchronous Preconditionner
+ Volume contact model



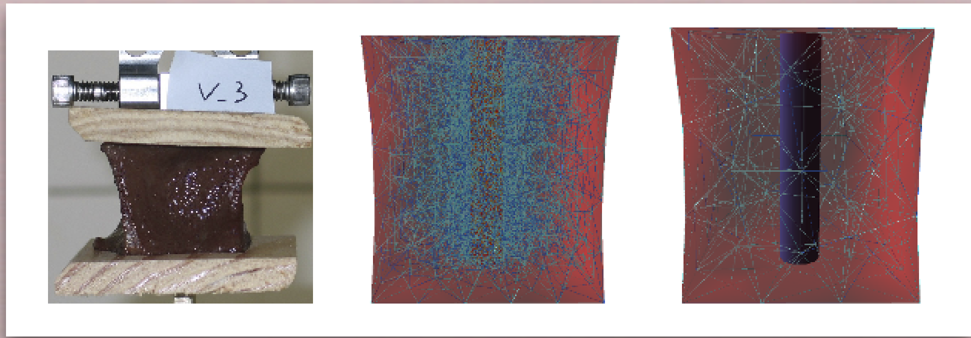
Asynchronous Preconditionner
+ Vascularized Liver
+ Constraint-based needle insertion

Courtecuisse et al. SIGGRAPH demo
Courtecuisse et al. MEDIA (submitted)

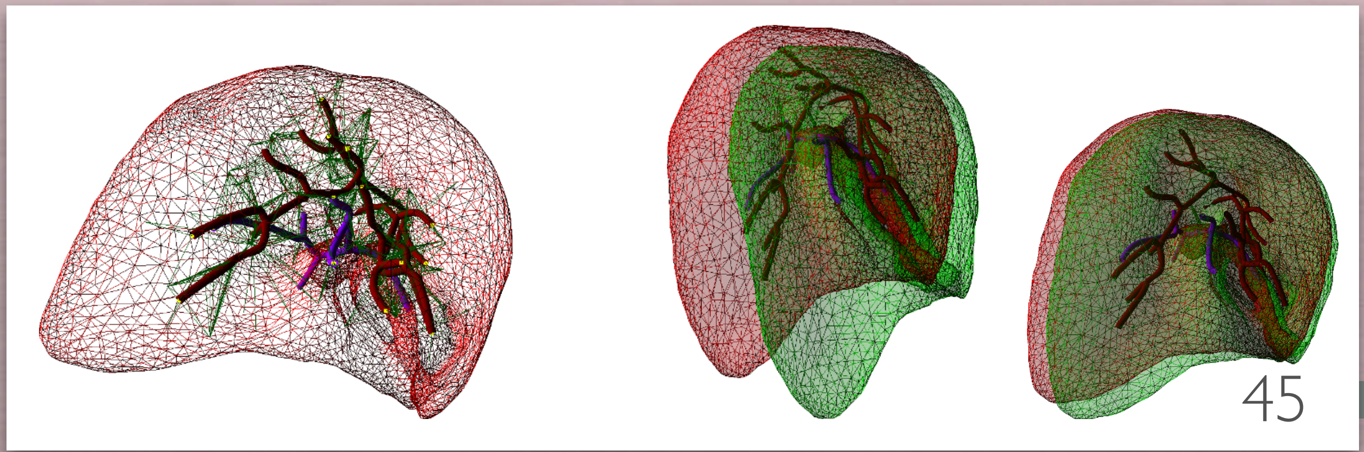
VASCULARIZED ORGAN MODEL



- Mechanical coupling between vessel and parenchyma
  - Corotational frame inside tetra elements
  - Coupled with the beam nodes (displacements & internal forces)
- *In vivo* measurement of vascularized sample response
  - Significant difference of stiffness with and without the vessel
  - Try to reproduce this difference numerically



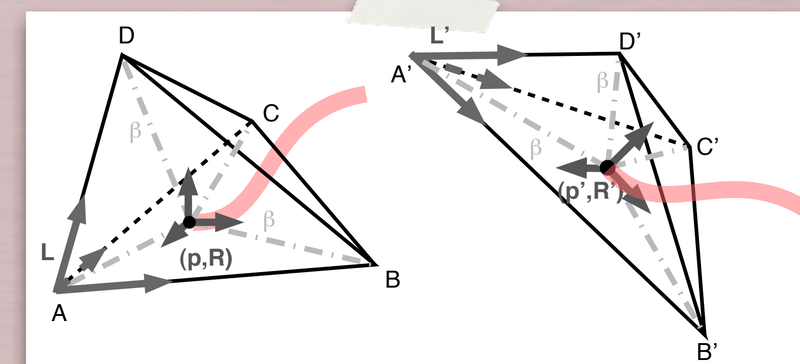
- Numerical Simulation of Vascularized Tissue
  - Numerical validation of the coupling (err <10% force response)
  - Simulation on liver model (60 FPS)
    - Parenchyma: 2620 tetra
    - Vessels: 314 beams

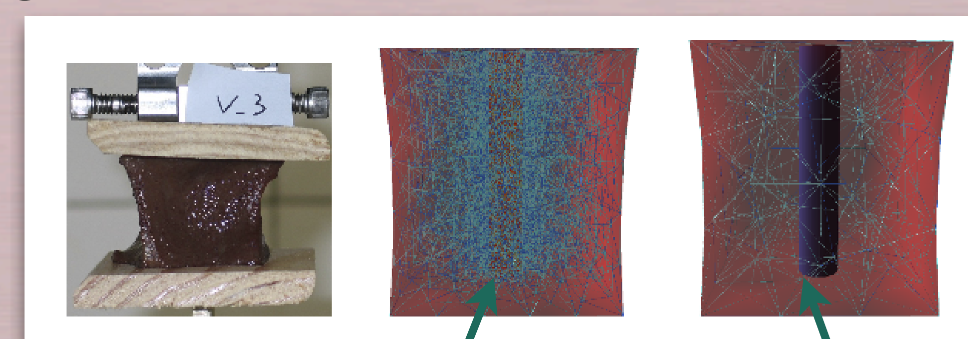


I. Peterlick et al. MICCAI 2012

VASCULARIZED ORGAN MODEL

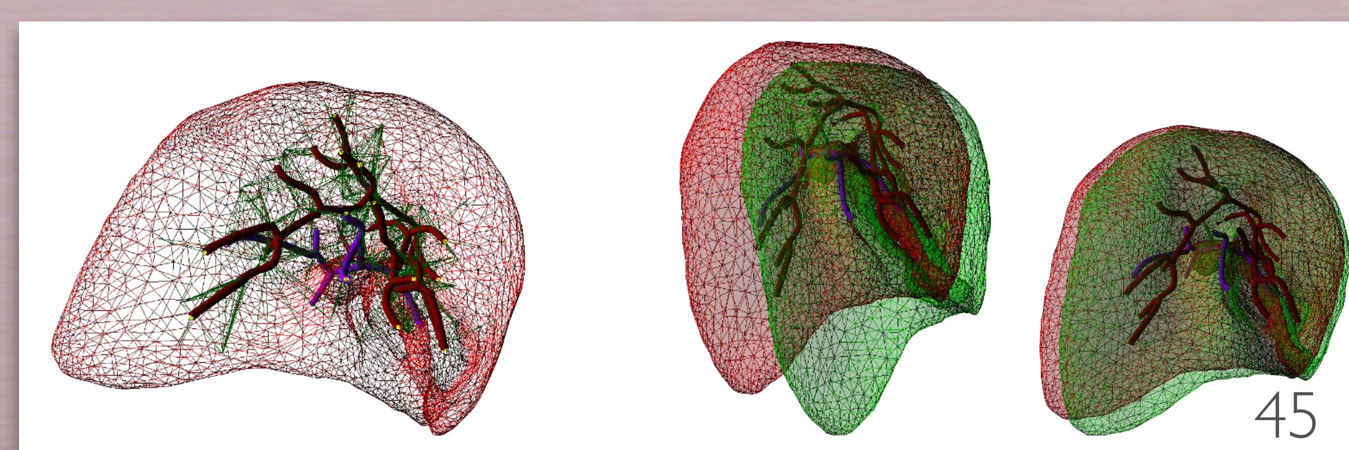
- Mechanical coupling between vessel and parenchyma
  - Corotational frame inside tetra elements
  - Coupled with the beam nodes (displacements & internal forces)
- *In vivo* measurement of vascularized sample response
  - Significant difference of stiffness with and without the vessel
  - Try to reproduce this difference numerically
- Numerical Simulation of Vascularized Tissue
  - Numerical validation of the coupling (err <10% force response)
  - Simulation on liver model (60 FPS)
    - Parenchyma: 2620 tetra
    - Vessels: 314 beams





52.000 Tetra

170 Tetra
2 beams



I. Peterlick et al. MICCAI 2012

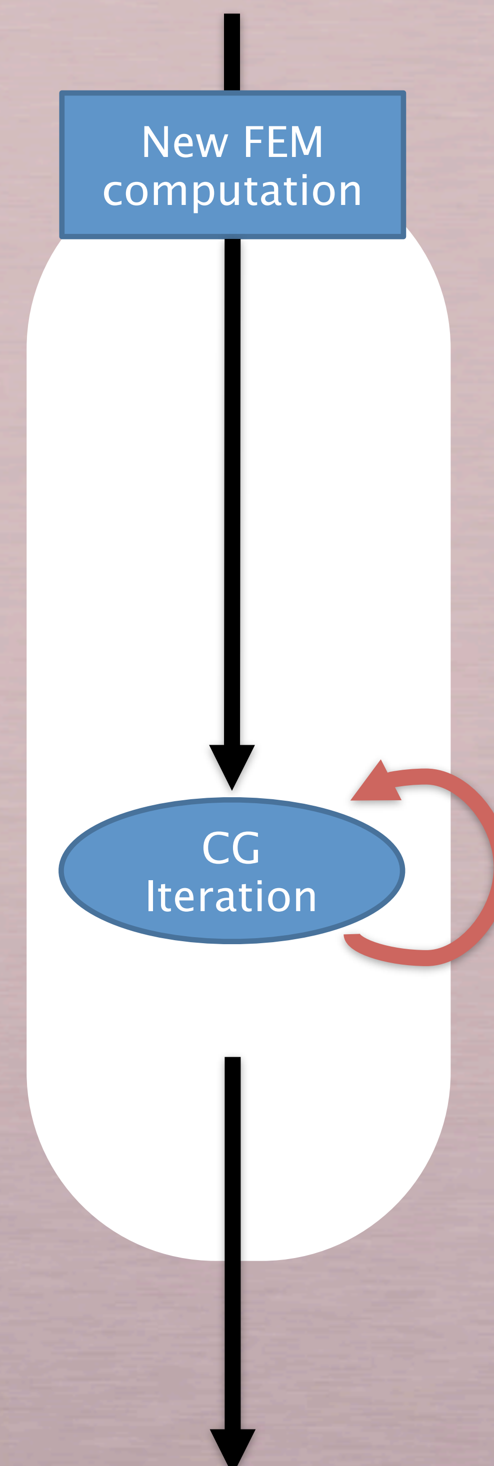
PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

Problématique:

- Grandes déformations = Modèles FEM non-linéaires
  - Linéarisation à chaque pas de la simulation
  - Un système $\mathbf{Ax} = \mathbf{b}$ à résoudre
  - Matrice **A** de grande taille mais creuse
- Hétérogénéités = Systèmes matriciels mal conditionnés
  - Mauvaise convergence algorithmes itératifs
  - Non compatible avec temps-réel

préconditionneur: $\mathbf{A}\ x = b$



Motivations pour le préconditionneur asynchrone:

- Cohérence temporelle de la valeur de **A**
- Factorisation d'une valeur «ancienne» de **A** pour **P**
- Le coût de la factorisation de **P** est déporté

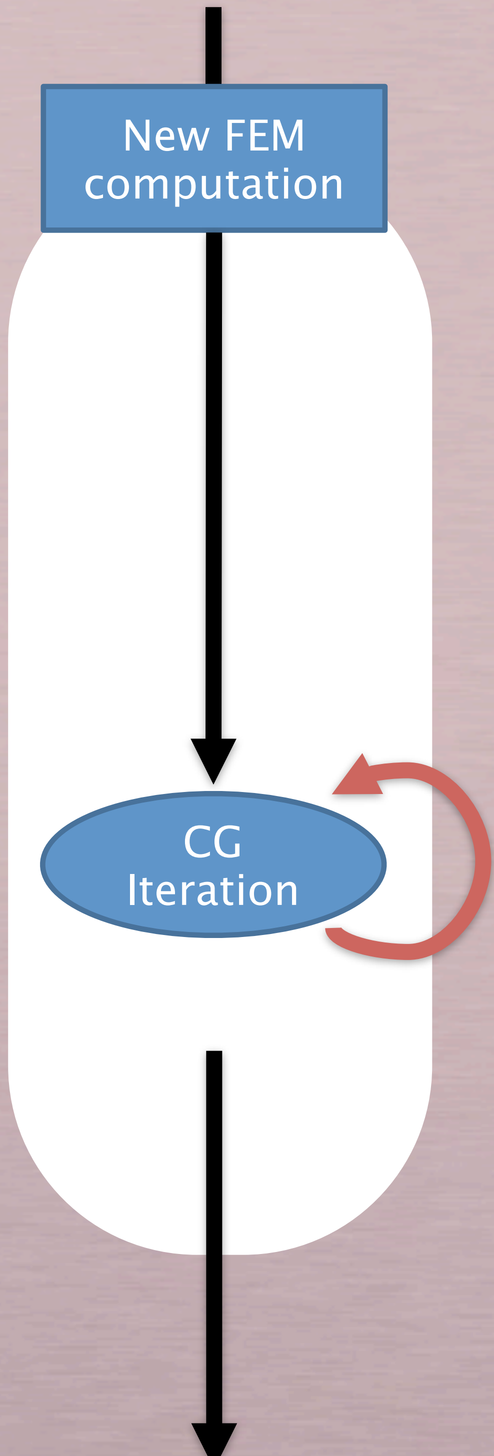
PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

Problématique:

- Grandes déformations = Modèles FEM non-linéaires
  - Linéarisation à chaque pas de la simulation
  - Un système **Ax** = **b** à résoudre
  - Matrice **A** de grande taille mais creuse
- Hétérogénéités = Systèmes matriciels mal conditionnés
  - Mauvaise convergence algorithmes itératifs
  - Non compatible avec temps-réel

préconditionneur: $\underbrace{\mathbf{P}^{-1}\mathbf{A}}_{\approx \mathbf{I}}\, x = \mathbf{P}^{-1} b$



Motivations pour le préconditionneur asynchrone:

- Cohérence temporelle de la valeur de **A**
- Factorisation d'une valeur «ancienne» de **A** pour **P**
- Le coût de la factorisation de **P** est déporté

PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

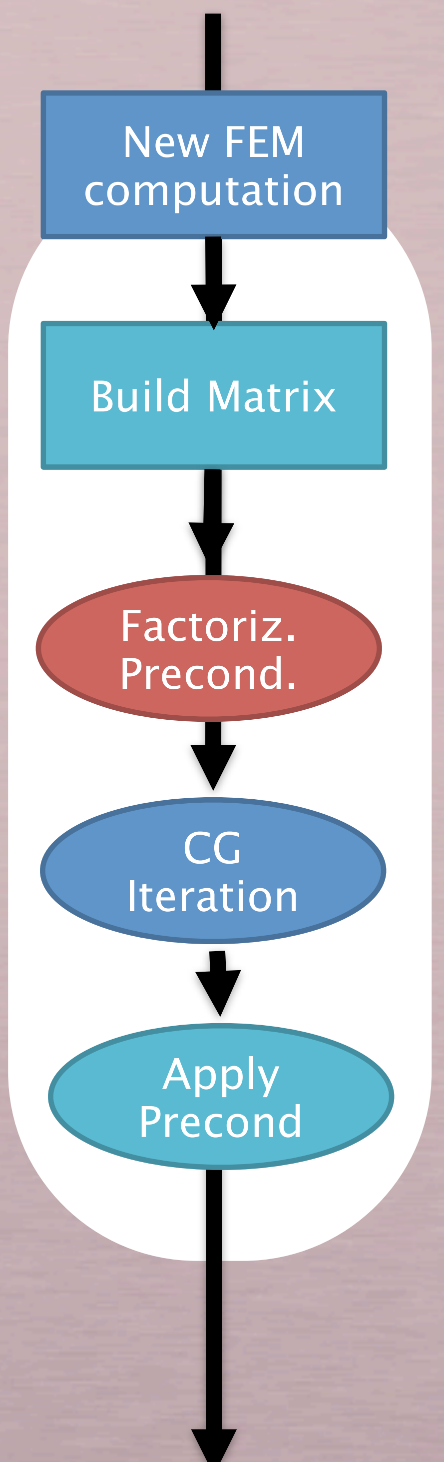
Problématique:

- Grandes déformations = Modèles FEM non-linéaires
  - Linéarisation à chaque pas de la simulation
  - Un système **Ax** = **b** à résoudre
  - Matrice **A** de grande taille mais creuse
- Hétérogénéités = Systèmes matriciels mal conditionnés
  - Mauvaise convergence algorithmes itératifs
  - Non compatible avec temps-réel

préconditionneur: $\underbrace{\mathbf{P}^{-1}\mathbf{A}}_{\approx \mathbf{I}}\, x = \mathbf{P}^{-1} b$

Motivations pour le préconditionneur asynchrone:

- Cohérence temporelle de la valeur de **A**
- Factorisation d'une valeur «ancienne» de **A** pour **P**
- Le coût de la factorisation de **P** est déporté



PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

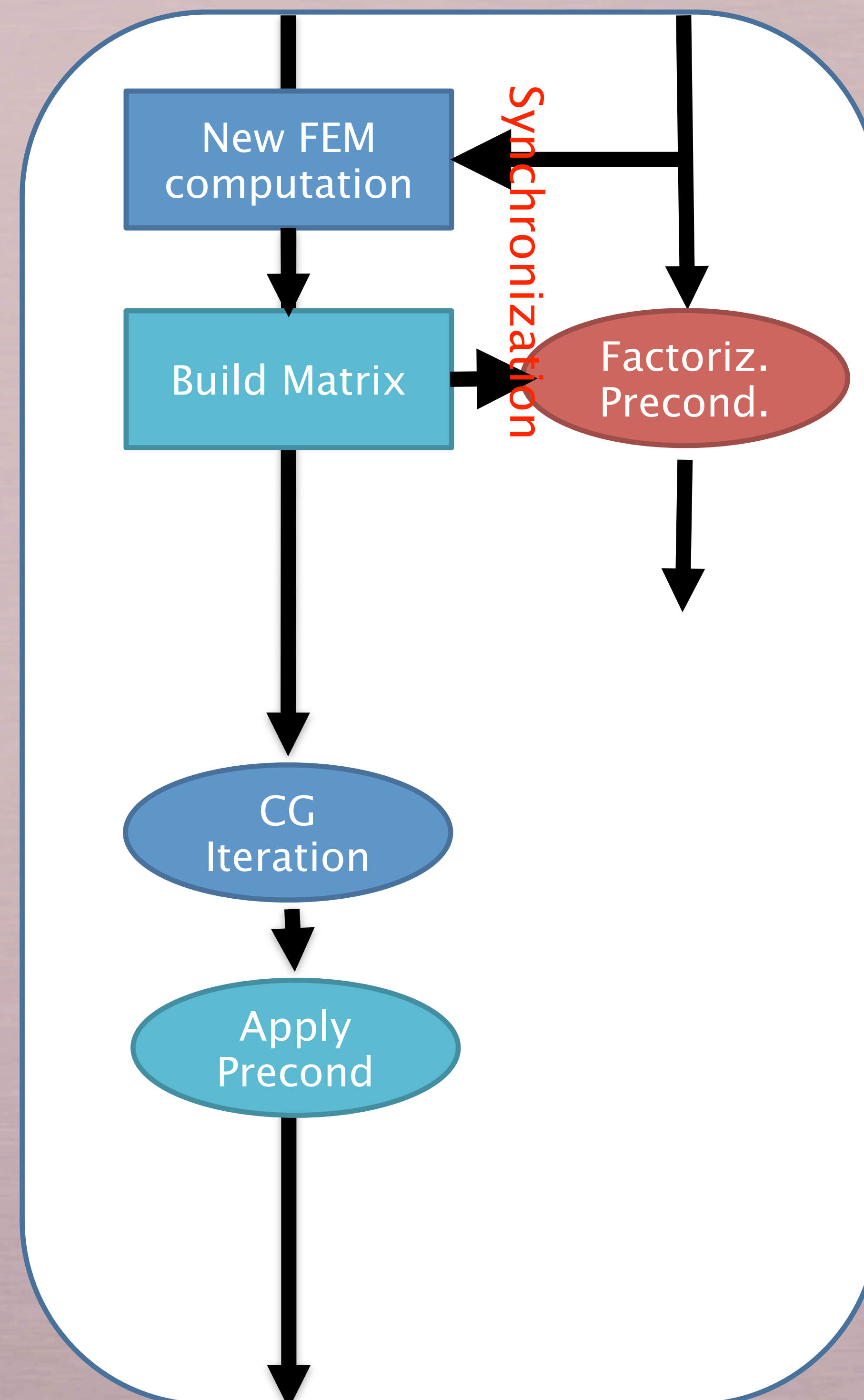
Problématique:

- Grandes déformations = Modèles FEM non-linéaires
    - Linéarisation à chaque pas de la simulation
    - Un système **Ax** = **b** à résoudre
    - Matrice **A** de grande taille mais creuse
- Hétérogénéités = Systèmes matriciels mal conditionnés
    - Mauvaise convergence algorithmes itératifs
    - Non compatible avec temps-réel

préconditionneur: $$\underbrace{\mathbf{P}^{-1}\mathbf{A}}_{\approx \mathbf{I}} x = \mathbf{P}^{-1} b$$

Motivations pour le préconditionneur asynchrone:

- Cohérence temporelle de la valeur de **A**
- Factorisation d'une valeur «ancienne» de **A** pour **P**
- Le coût de la factorisation de **P** est déporté



Real time simulation with semi-Implicit time integration

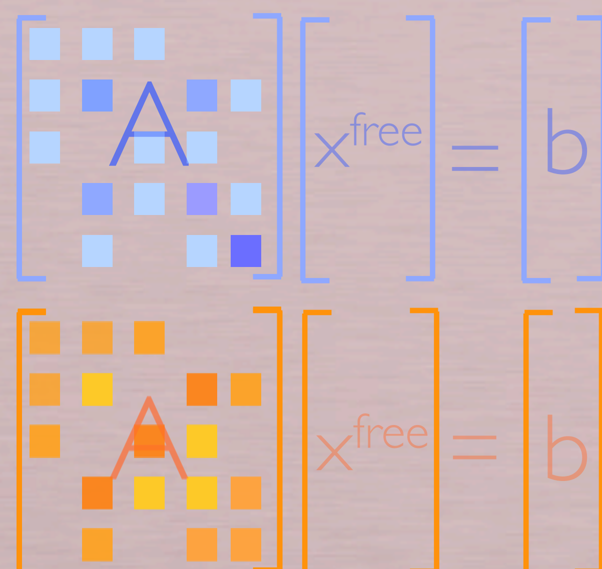
We use an implicit time integration which enables large time steps, but requires to solve a linear system $Ax = b$ at each time step

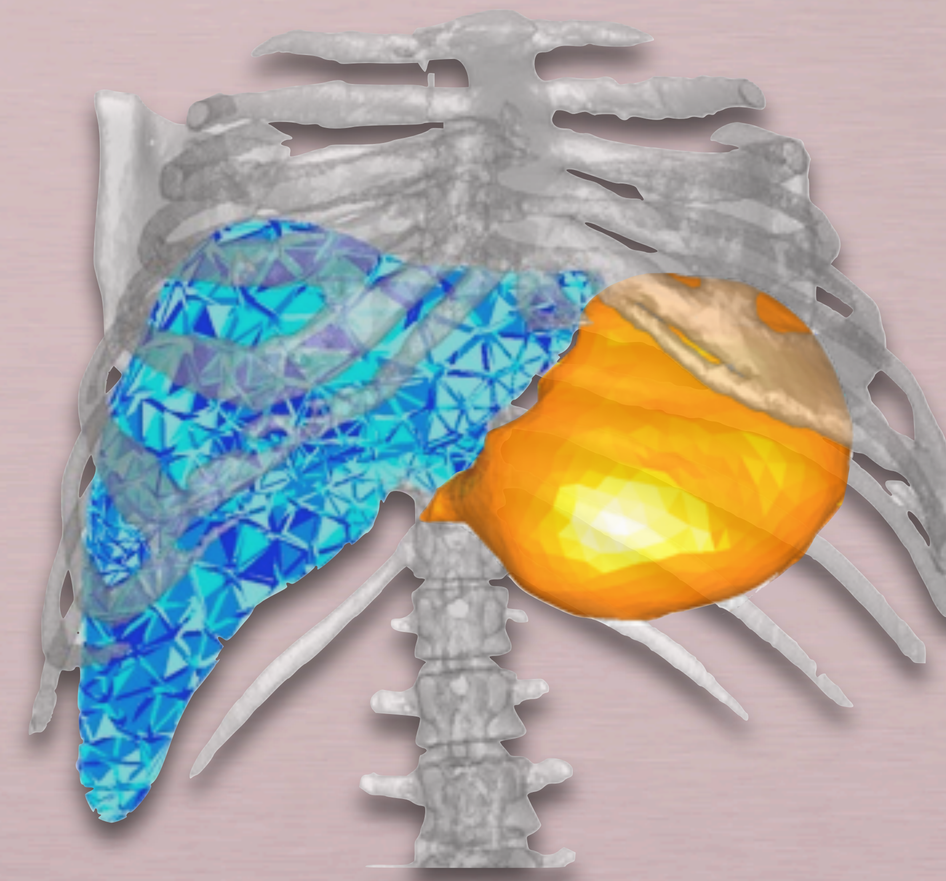
PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

Résolution des contraintes de contact:

- Mouvement Libre
- Formulation des contraintes
- Résolution
- Correction $x = x^{free} + Dx$





PRÉCONDITIONNEUR ASYNCHRONE

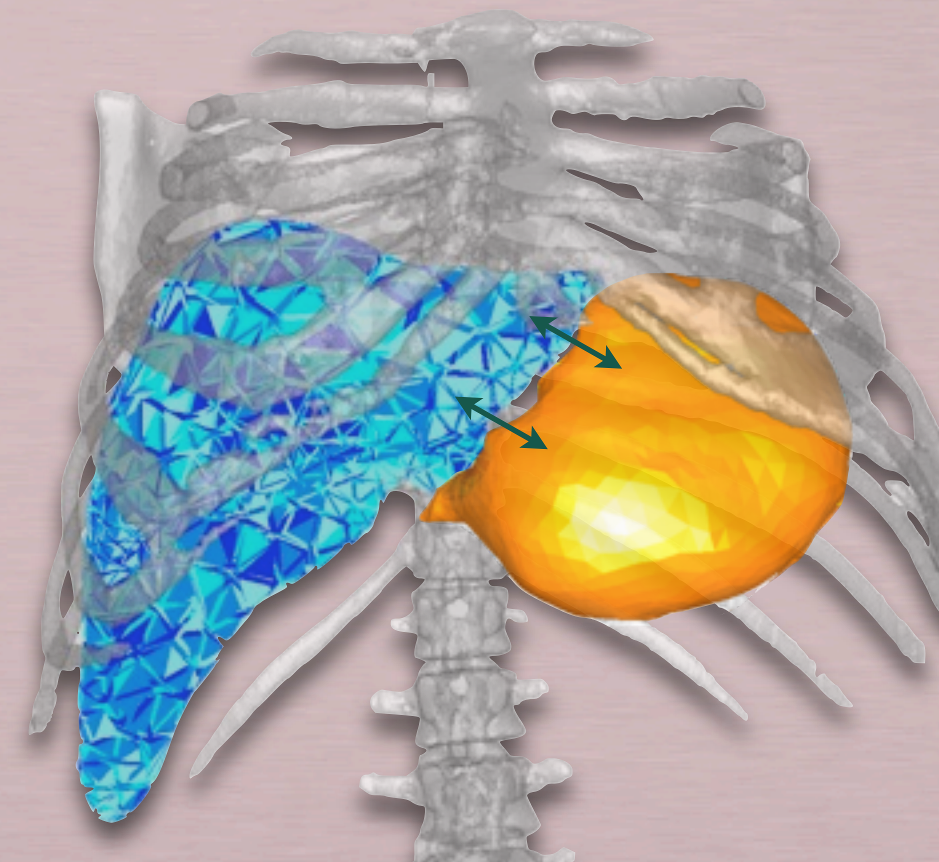
Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

Résolution des contraintes de contact:

- Mouvement Libre
- Formulation des contraintes
- Résolution
- Correction $x = x^{free} + Dx$

$$[A][x] = [b] + [H^T][\lambda]$$

$$[A][x] = [b] + [H^T][\lambda]$$

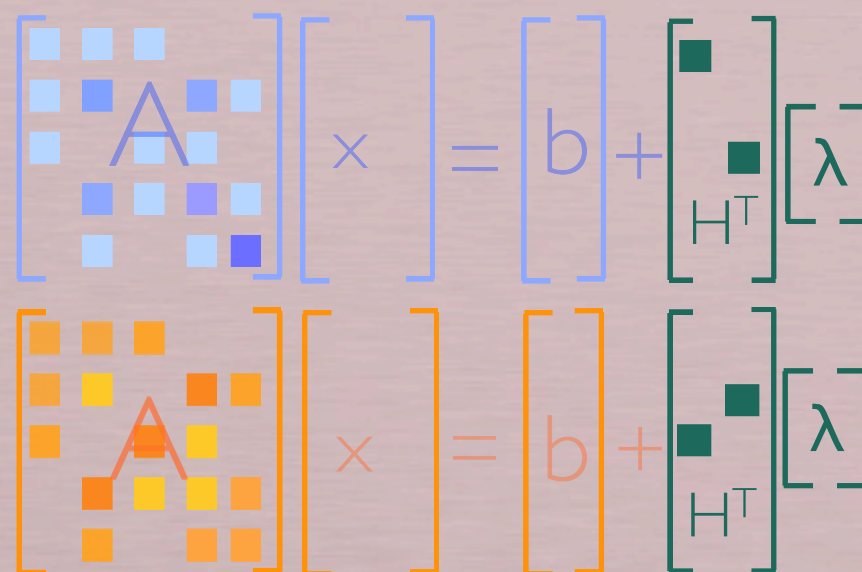


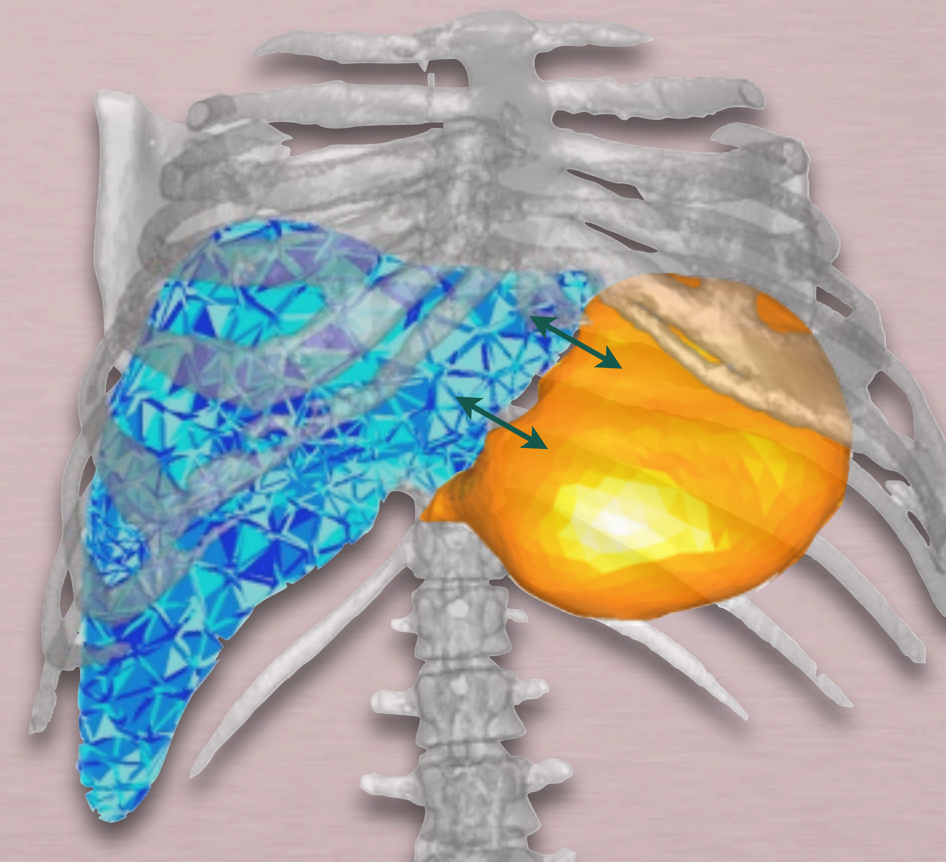
PRÉCONDITIONNEUR ASYNCHRONE

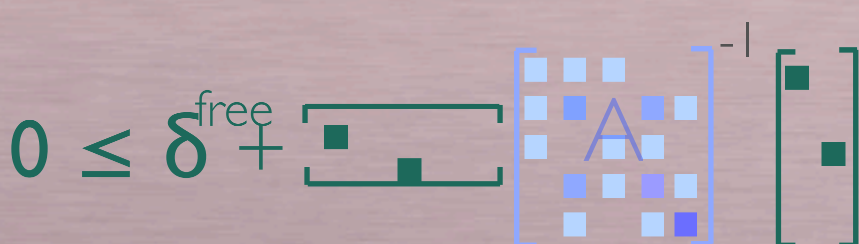
Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

Résolution des contraintes de contact:

- Mouvement Libre
- Formulation des contraintes
- Résolution
- Correction $x = x^{free} + Dx$





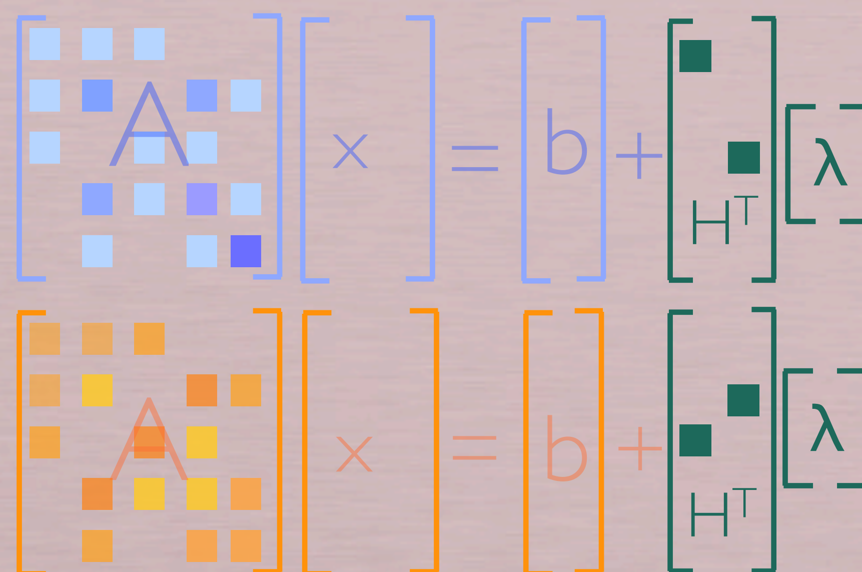


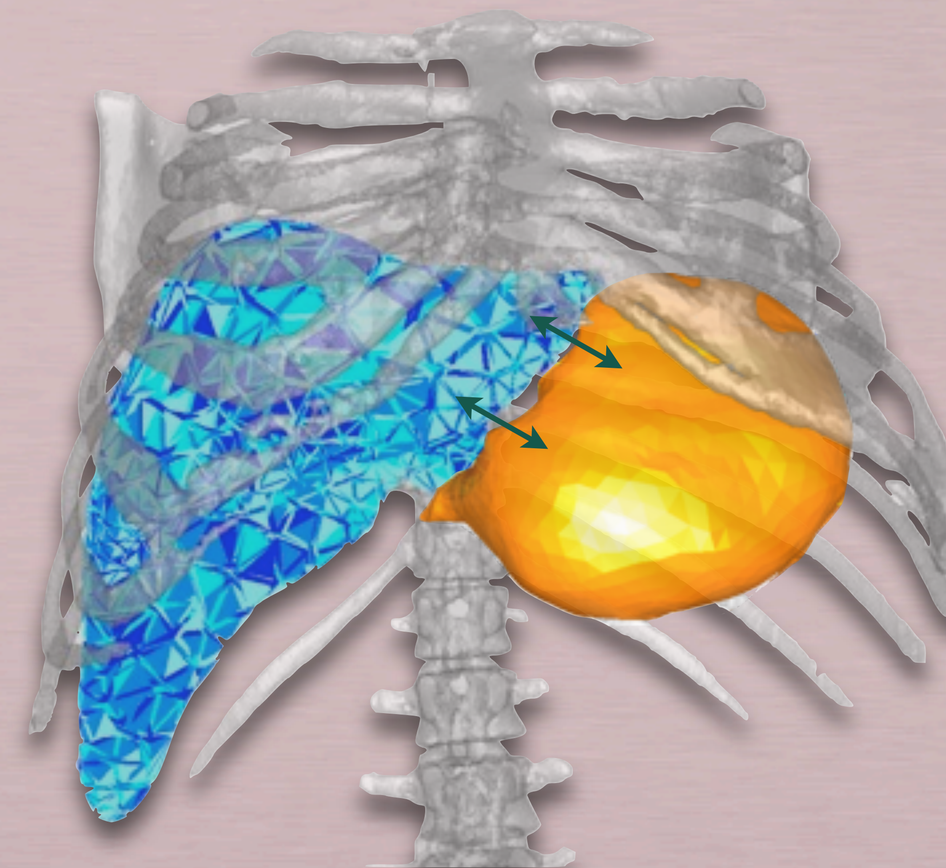
PRÉCONDITIONNEUR ASYNCHRONE

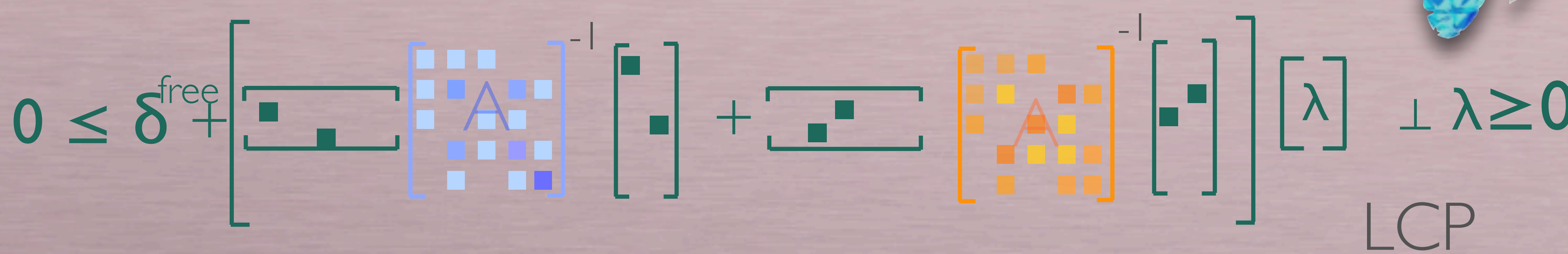
Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

Résolution des contraintes de contact:

- Mouvement Libre
- Formulation des contraintes
- Résolution
- Correction $x = x^{free} + Dx$





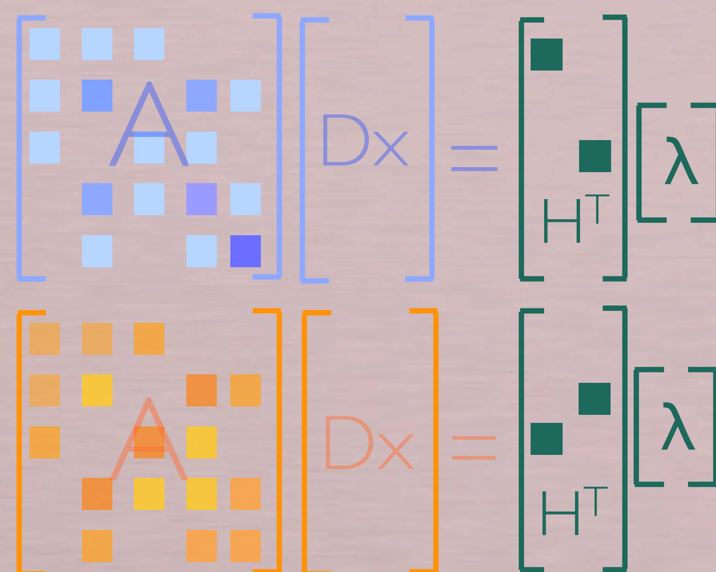


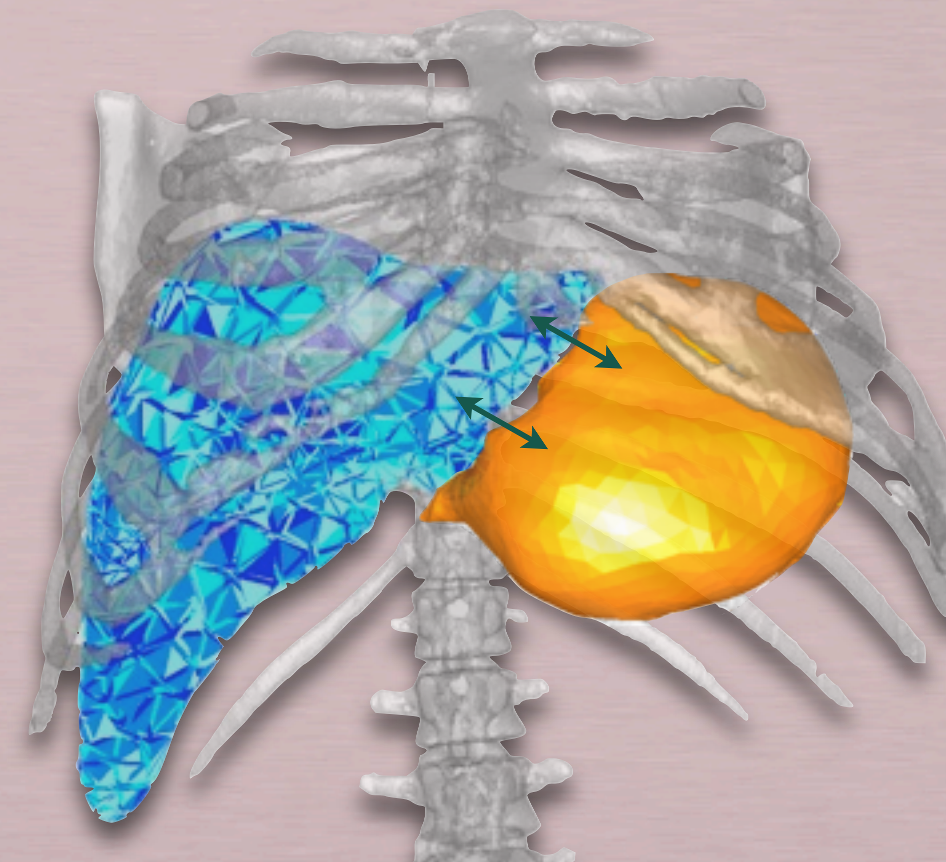
PRÉCONDITIONNEUR ASYNCHRONE

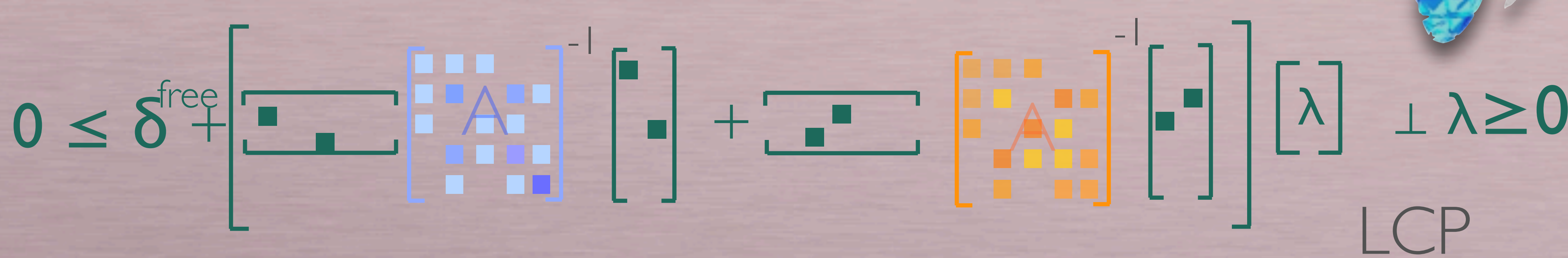
Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

Résolution des contraintes de contact:

- Mouvement Libre
- Formulation des contraintes
- Résolution
- Correction $x = x^{free} + Dx$





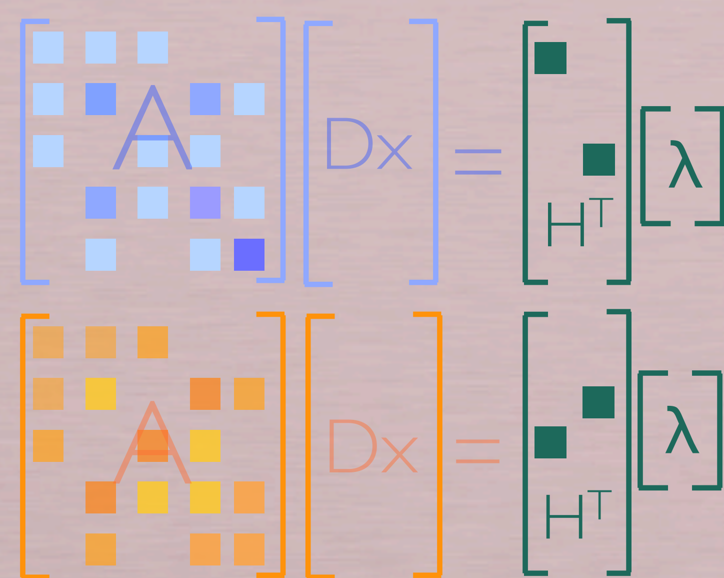


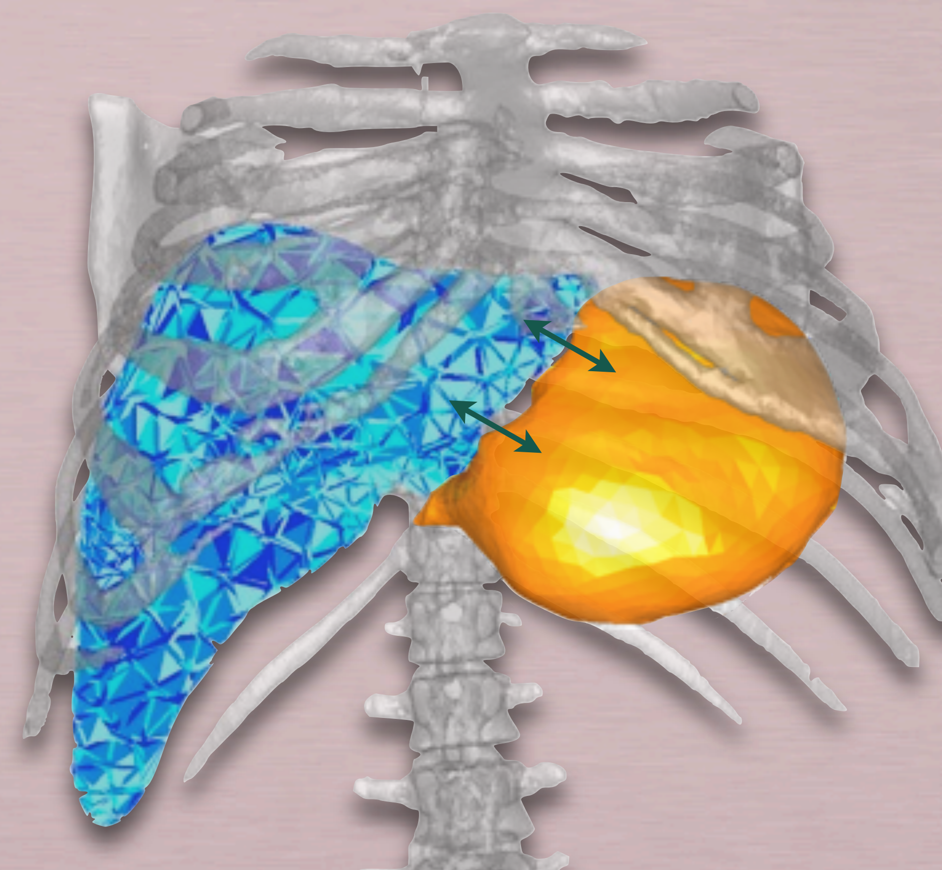
PRÉCONDITIONNEUR ASYNCHRONE

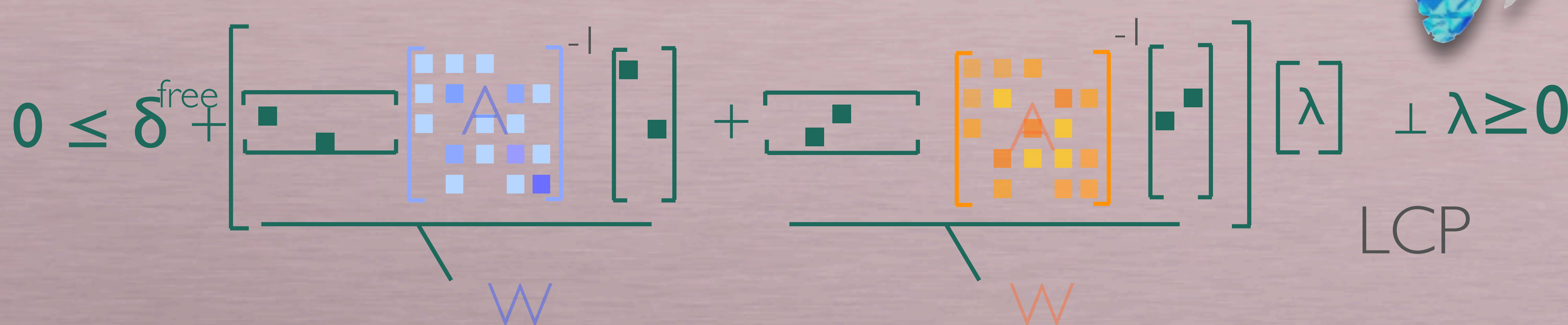
Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

Résolution des contraintes de contact:

- Mouvement Libre
- Formulation des contraintes
- Résolution
- Correction $x = x^{free} + Dx$







[Compliance dans l'espace des constraintes]

PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} = \mathbf{H}\,\mathbf{A}^{-1}\,\mathbf{H}^T$$

1. Comme on a un très bon préconditionneur, on a une très bonne approximation de la matrice du LCP
2. Préconditionneur appliqué sur chaque colonne de H^T
   On obtient: $S = (LDL)^{-1}H^T$
3. Matrice de compliance par multiplication $H\,S$
   On obtient: $W = H\,(LDL)^{-1}H^T$

Courtecuisse et al. MICCAI 2011

PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} \approx \mathbf{H}\,(\mathbf{LDL})^{-1}\mathbf{H}^{T}$$

1. Comme on a un très bon préconditionneur, on a une très bonne approximation de la matrice du LCP
2. Préconditionneur appliqué sur chaque colonne de H^T
   On obtient: $S = (LDL)^{-1}H^T$
3. Matrice de compliance par multiplication H S
   On obtient: $W = H\,(LDL)^{-1}H^T$

Courtecuisse et al. MICCAI 2011

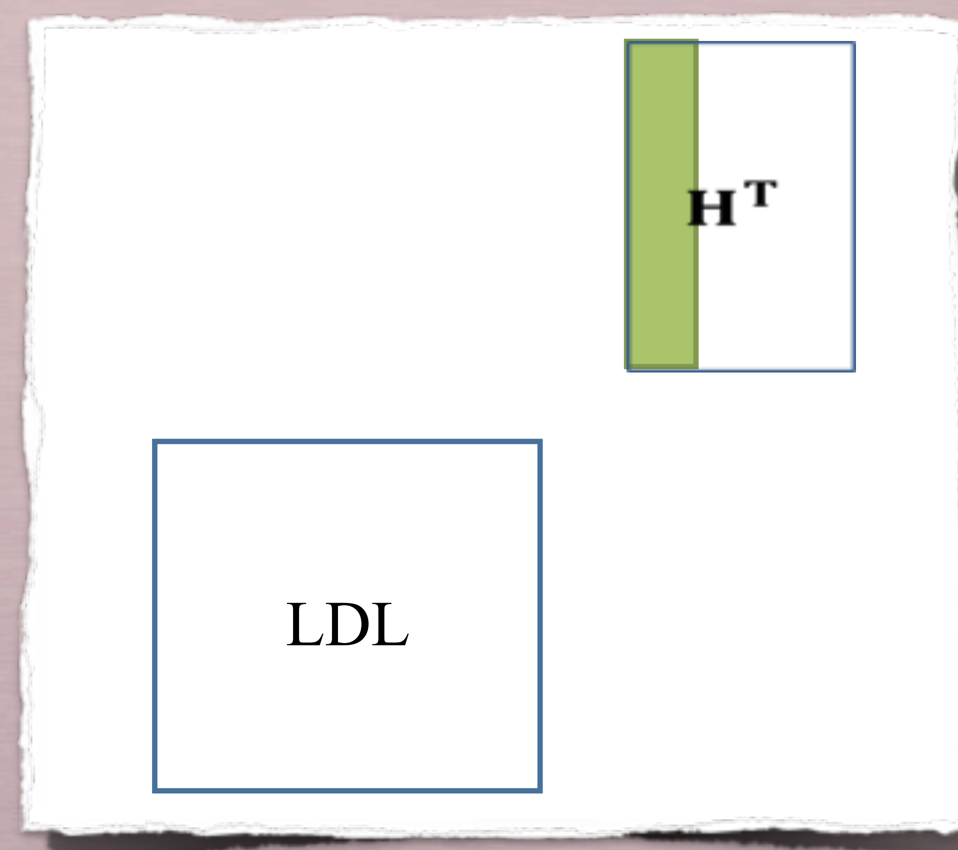
PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} \approx \mathbf{H}\,(\mathbf{LDL})^{-1}\mathbf{H}^{\mathbf{T}}$$

1. Comme on a un très bon préconditionneur, on a une très bonne approximation de la matrice du LCP
2. Préconditionneur appliqué sur chaque colonne de H^T
   On obtient: $S = (LDL)^{-1}H^T$
3. Matrice de compliance par multiplication $H\ S$
   On obtient: $W = H\ (LDL)^{-1}H^T$



Courtecuisse et al. MICCAI 2011

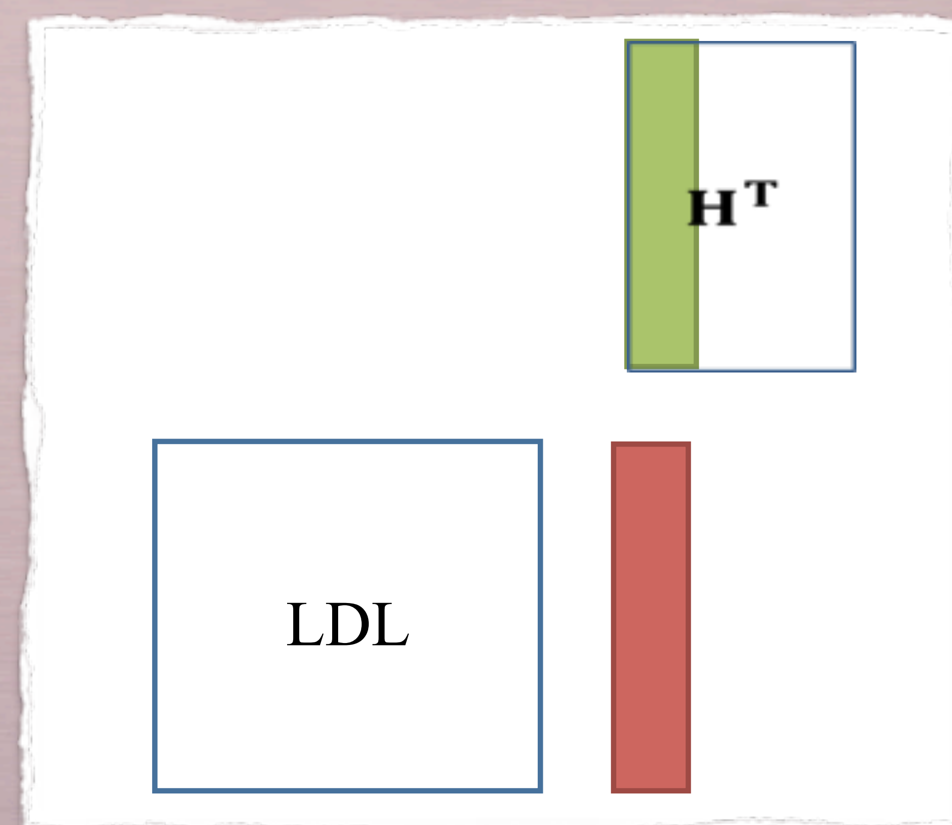
PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} \approx \mathbf{H}\,(\mathbf{LDL})^{-1}\mathbf{H}^{T}$$

1. Comme on a un très bon préconditionneur, on a une très bonne approximation de la matrice du LCP
2. Préconditionneur appliqué sur chaque colonne de H^T
   On obtient: $S = (LDL)^{-1}H^T$
3. Matrice de compliance par multiplication $H\,S$
   On obtient: $W = H\,(LDL)^{-1}H^T$



Courtecuisse et al. MICCAI 2011

PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

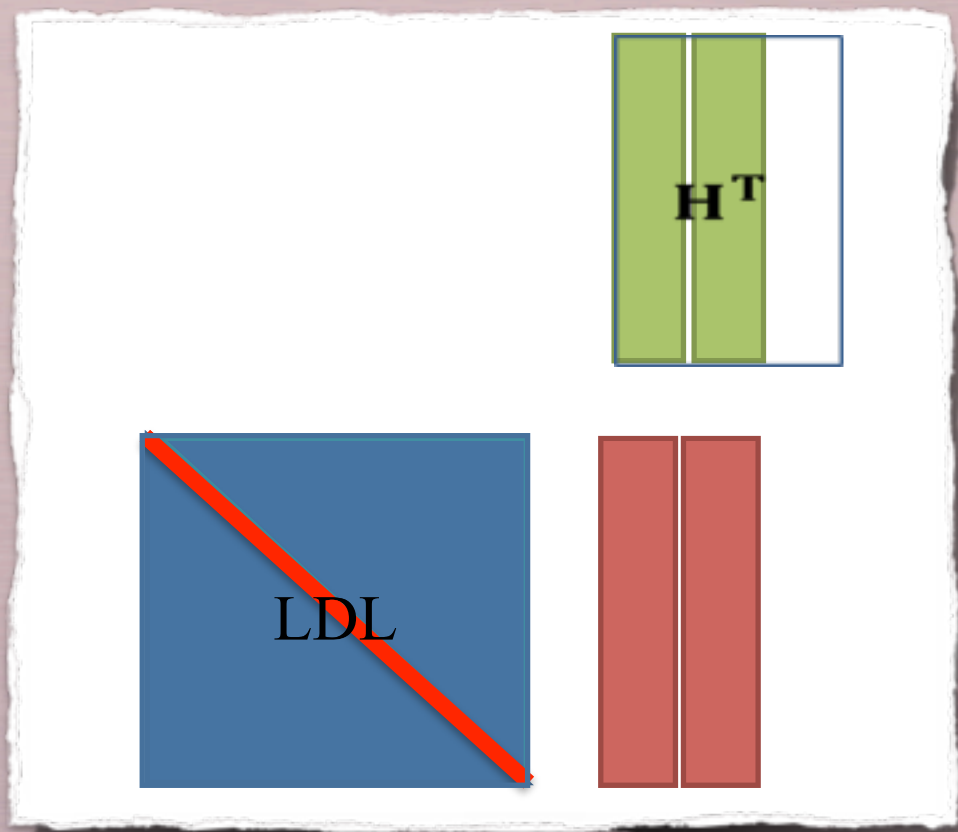
On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} \approx \mathbf{H}\,(\mathbf{LDL})^{-1}\mathbf{H}^{T}$$

1. Comme on a un très bon préconditionneur, on a une très bonne approximation de la matrice du LCP

2. Préconditionneur appliqué sur chaque colonne de H^T
   On obtient: $S = (LDL)^{-1}H^T$

3. Matrice de compliance par multiplication H S
   On obtient: $W = H\,(LDL)^{-1}H^T$



Courtecuisse et al. MICCAI 2011

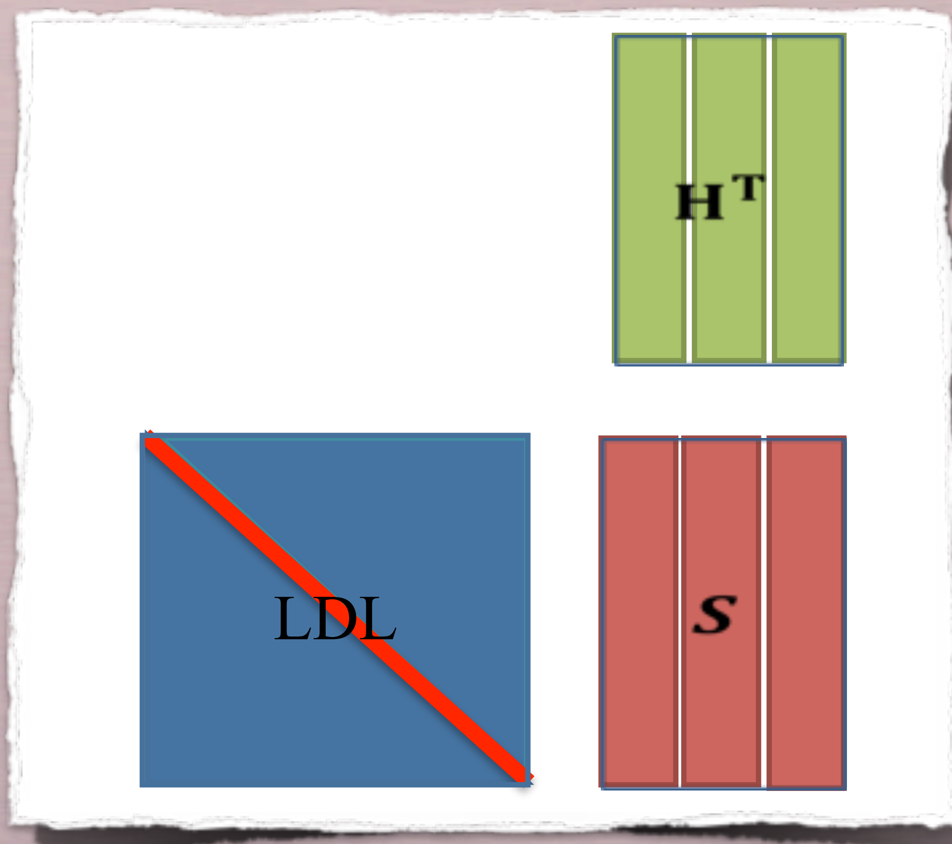
PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} \approx \mathbf{H}\,(\mathbf{LDL})^{-1}\mathbf{H}^{T}$$

1. Comme on a un très bon préconditionneur, on a une très bonne approximation de la matrice du LCP
2. Préconditionneur appliqué sur chaque colonne de H^T
   On obtient: $S = (LDL)^{-1}H^T$
3. Matrice de compliance par multiplication $H\,S$
   On obtient: $W = H\,(LDL)^{-1}H^T$



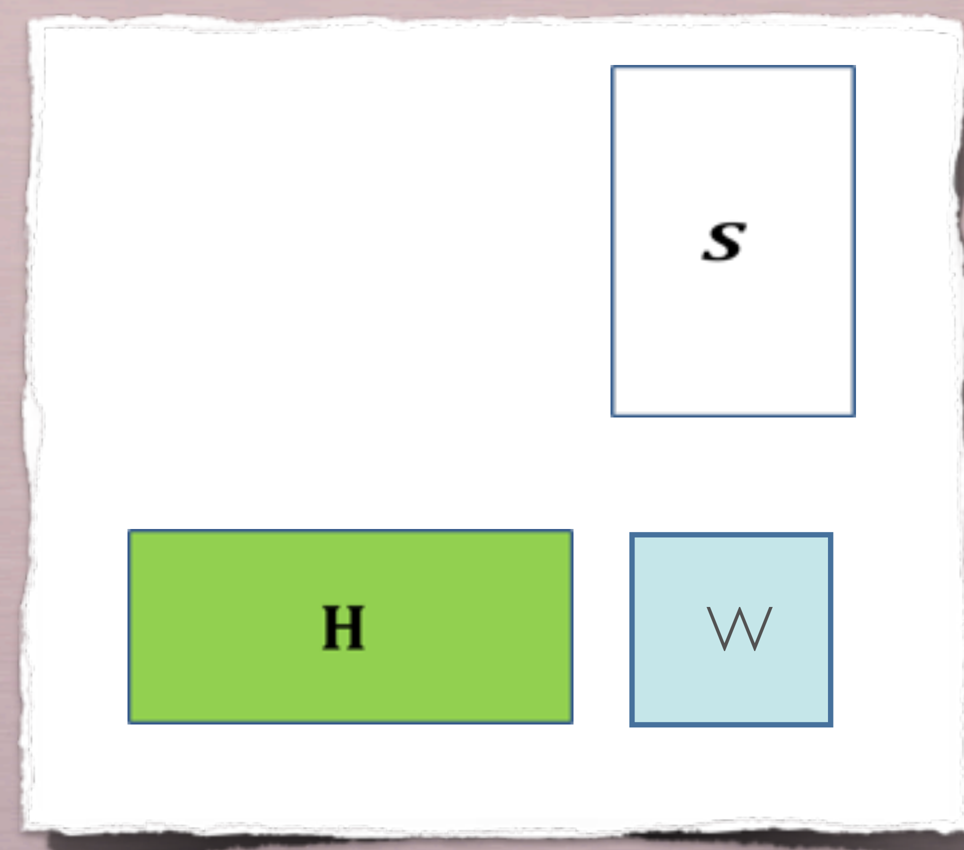
Courtecuisse et al. MICCAI 2011

PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} \approx \mathbf{H}\,(\mathbf{LDL})^{-1}\mathbf{H}^{T}$$



1. Comme on a un très bon préconditionneur, on a une très bonne approximation de la matrice du LCP
2. Préconditionneur appliqué sur chaque colonne de H^T
   On obtient: $S = (LDL)^{-1}H^T$
3. Matrice de compliance par multiplication H S
   On obtient: $W = H\,(LDL)^{-1}H^T$

Courtecuisse et al. MICCAI 2011

PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\boxed{\mathbf{W} = \mathbf{H}\,\mathbf{A}^{-1}\,\mathbf{H}^{T}}$$

Parallélisation:

- Premier niveau: Résolution séparée pour chaque contrainte
- Deuxième niveau: Application de la factorisation LDL

PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} \approx \mathbf{H}\,(\mathbf{LDL})^{-1}\mathbf{H}^{T}$$

Parallélisation:

- Premier niveau: Résolution séparée pour chaque contrainte
- Deuxième niveau: Application de la factorisation LDL

PRÉCONDITIONNEUR ASYNCHRONE

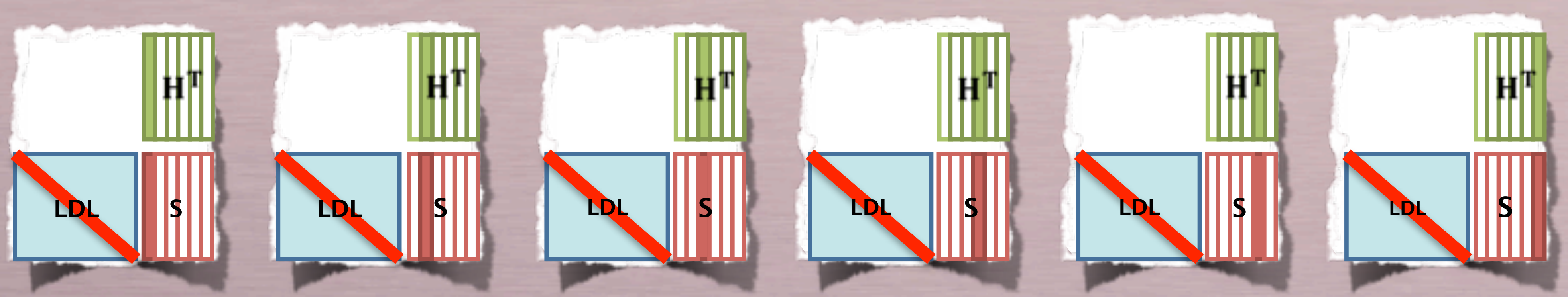
Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} \approx \mathbf{H}\,(\mathbf{LDL})^{-1}\mathbf{H}^T$$

Parallélisation:

- Premier niveau: Résolution séparée pour chaque contrainte
- Deuxième niveau: Application de la factorisation LDL



PRÉCONDITIONNEUR ASYNCHRONE

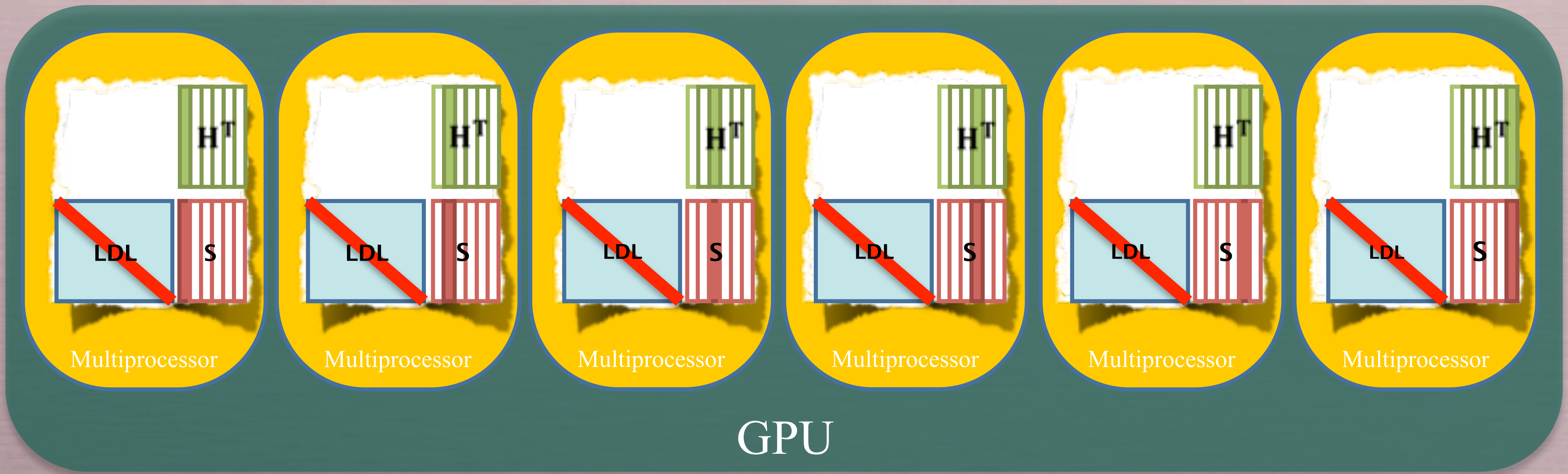
Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} \approx \mathbf{H}\,(\mathbf{LDL})^{-1}\mathbf{H}^T$$

Parallélisation:

- Premier niveau: Résolution séparée pour chaque contrainte
- Deuxième niveau: Application de la factorisation LDL



PRÉCONDITIONNEUR ASYNCHRONE

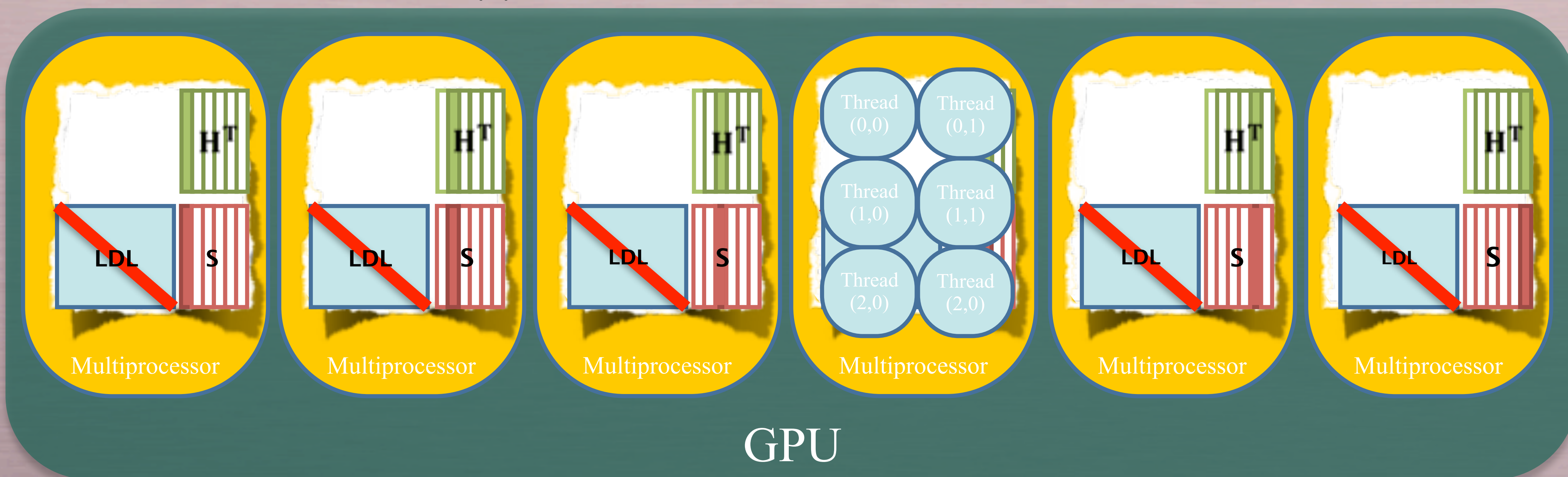
Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

On utilise le préconditionneur asynchrone pour approximer la matrice inverse:

$$\mathbf{W} \approx \mathbf{H}\,(\mathbf{LDL})^{-1}\mathbf{H}^T$$

Parallélisation:

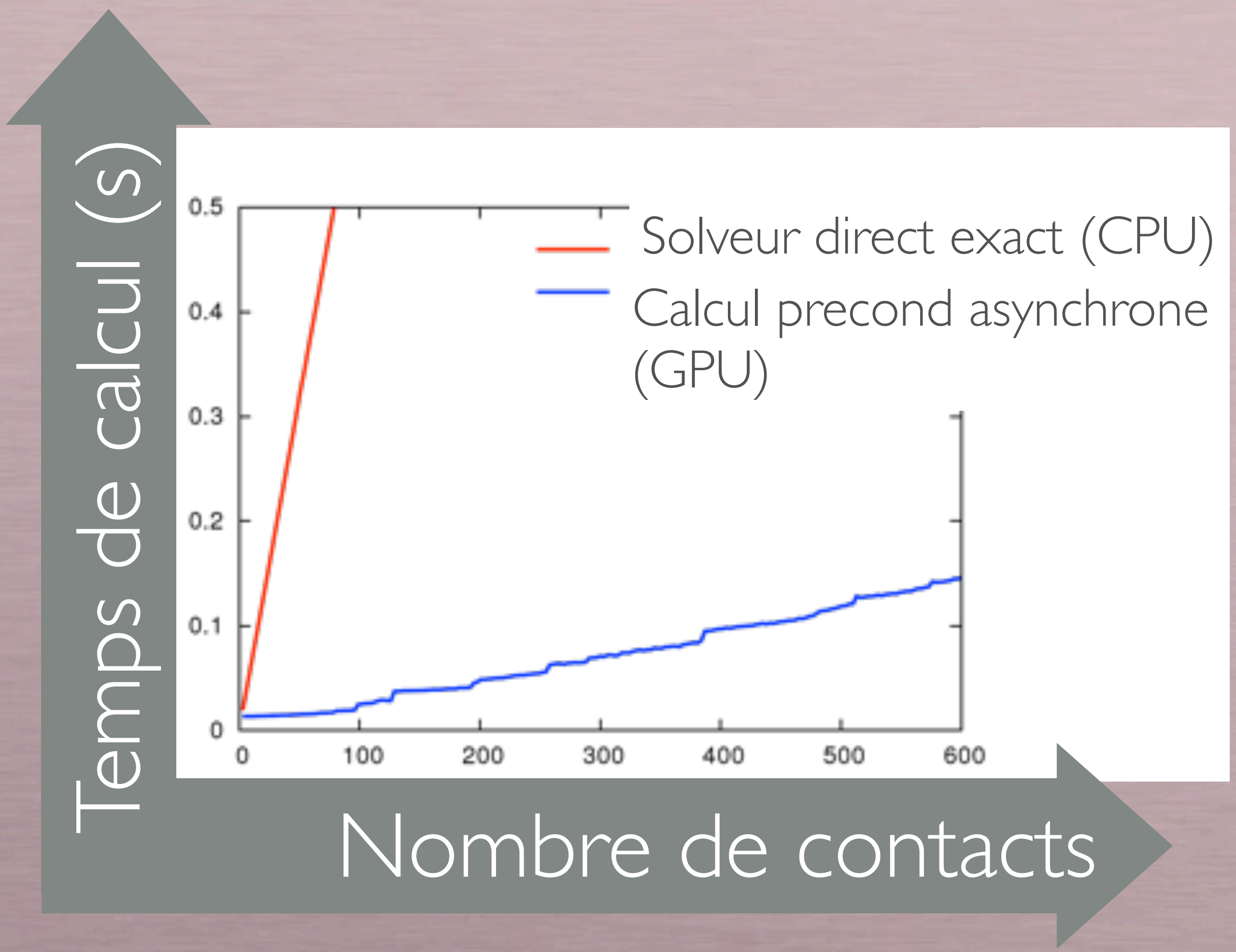
- Premier niveau: Résolution séparée pour chaque contrainte
- Deuxième niveau: Application de la factorisation LDL



PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

- Contacts sur des objets avec propriétés mécaniques hétérogènes
- Validation par comparaison avec la solution exacte
- Temps de calcul très réduit par rapport au calcul exact sur CPU
- [MICCAI 2011] [Journal Medical Image Analysis 2013]
- Transfert: Licence du module de calcul pour la start-up InSimo

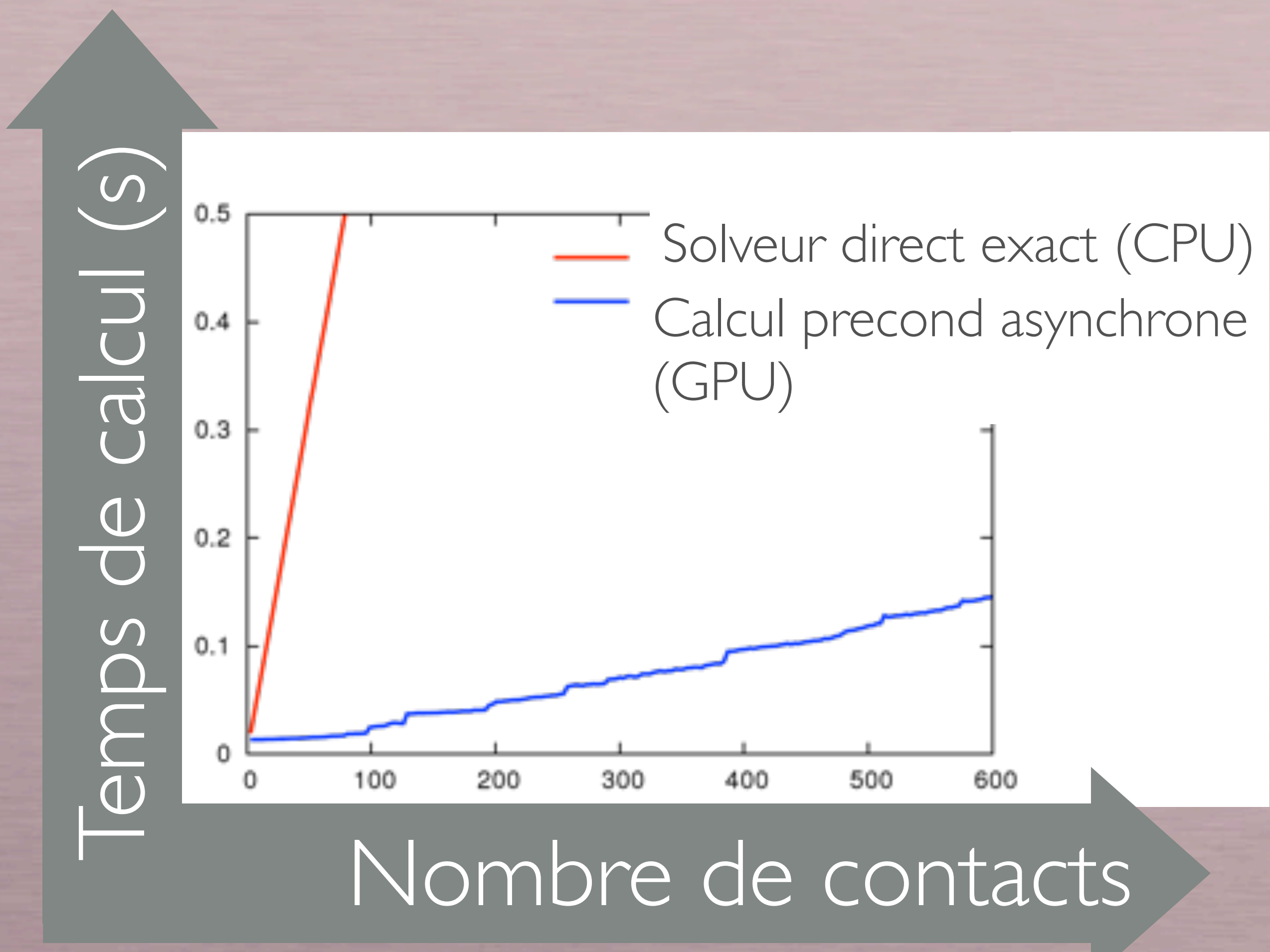


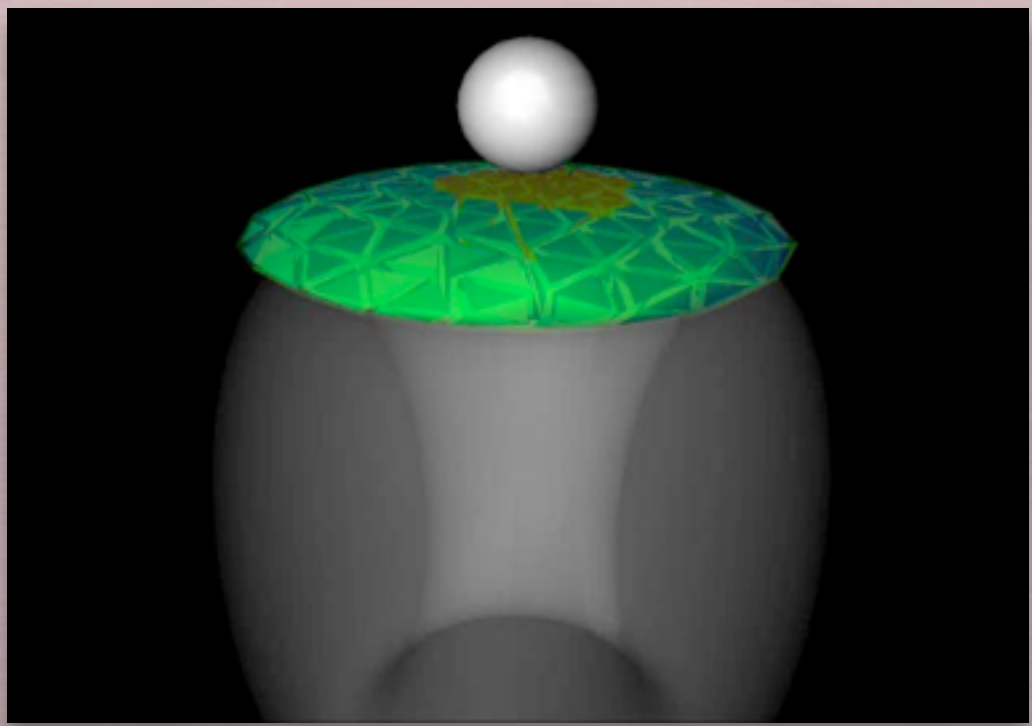
Vert: solution exacte
Bleu/rouge: notre méthode

PRÉCONDITIONNEUR ASYNCHRONE

Calcul temps-réel de modèles FEM hétérogènes et de la compliance des contraintes

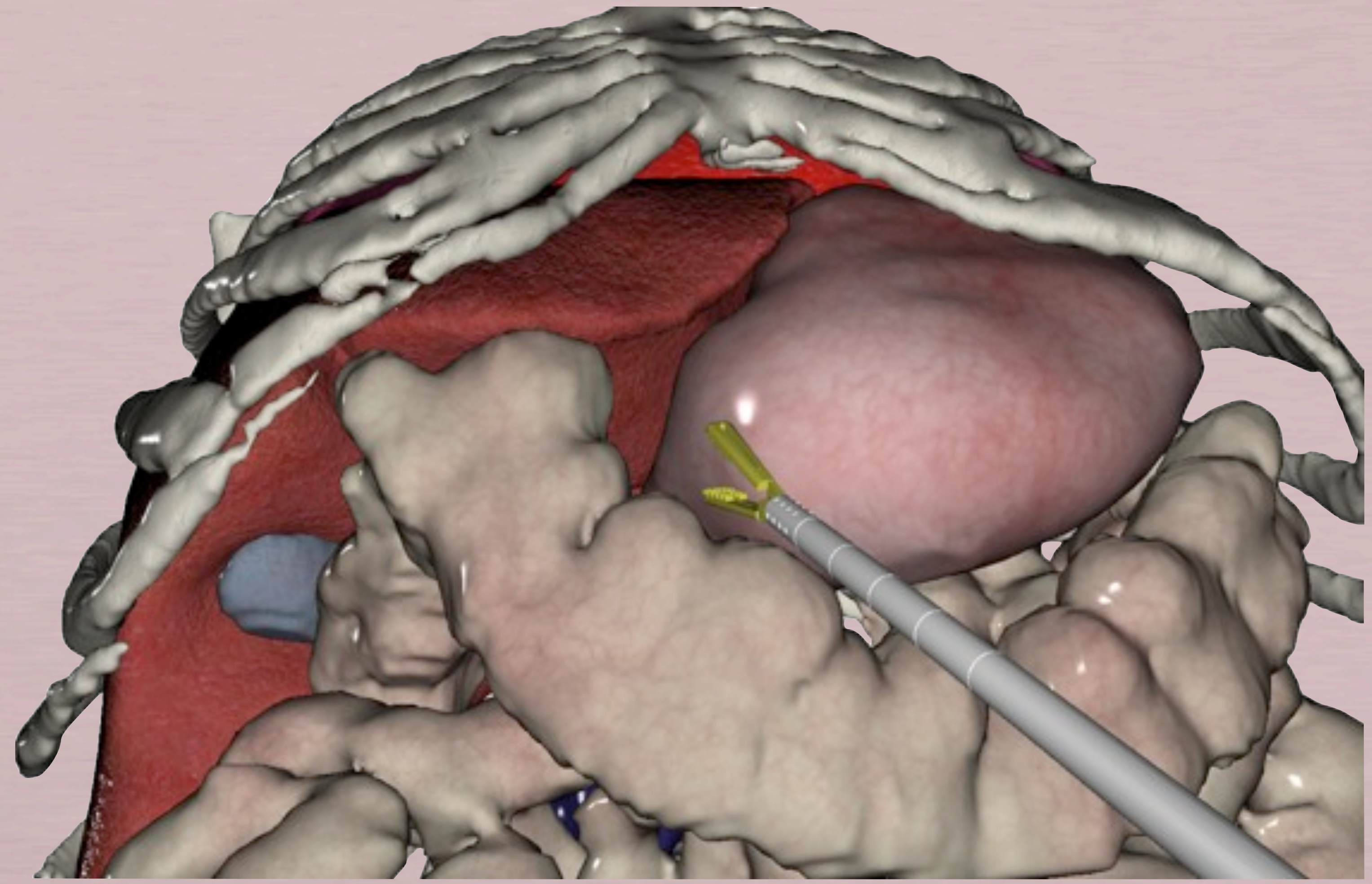
- Contacts sur des objets avec propriétés mécaniques hétérogènes
- Validation par comparaison avec la solution exacte
- Temps de calcul très réduit par rapport au calcul exact sur CPU
- [MICCAI 2011] [Journal Medical Image Analysis 2013]
- Transfert: Licence du module de calcul pour la start-up InSimo





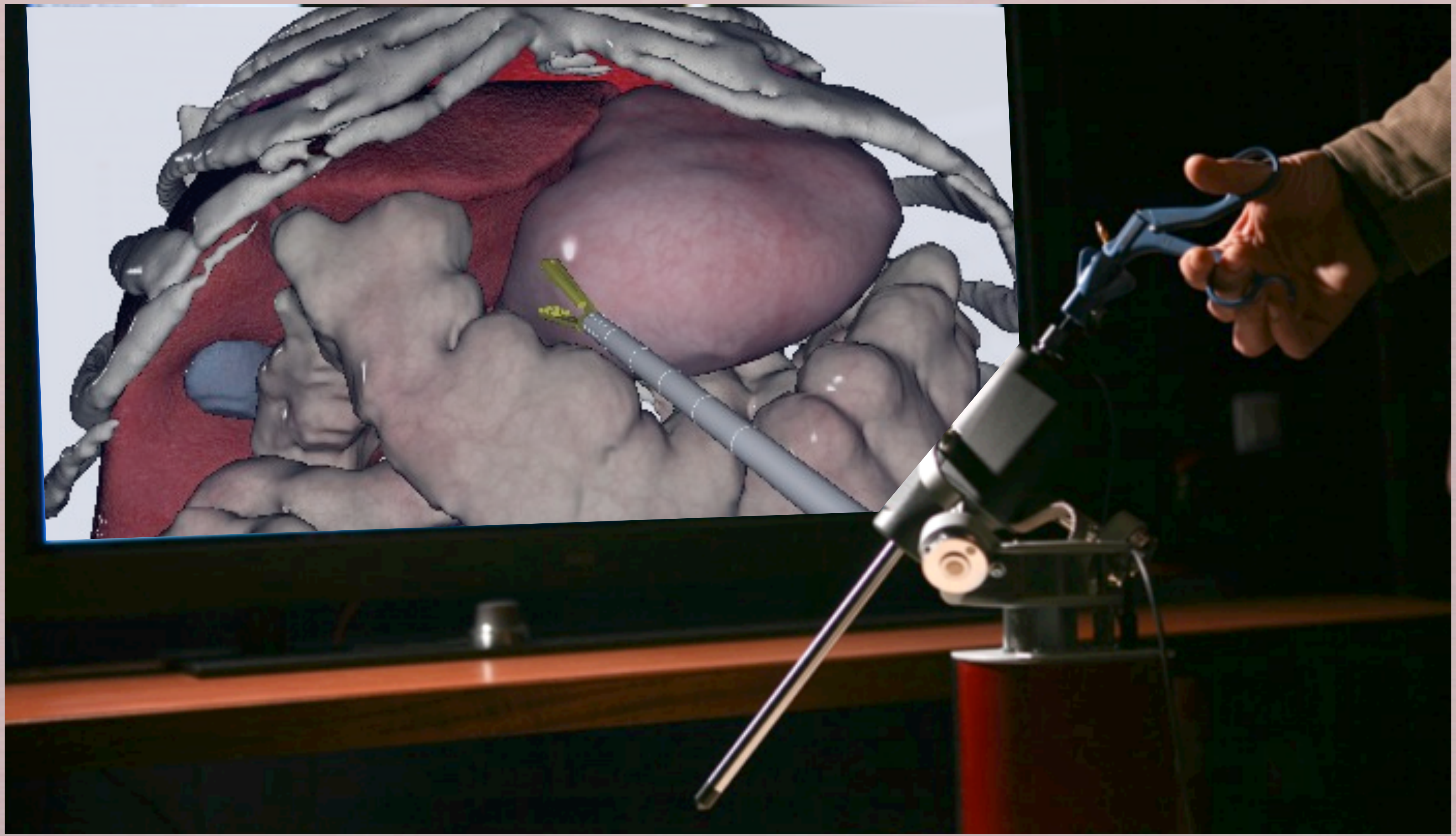
Vert: solution exacte
Bleu/rouge: notre méthode

- Haptic rendering and multithreading approaches
- Applications, ongoing research projects
- Perspective and Conclusion



HAPTIC RENDERING AND MULTITHREADING APPROACHES

- Haptic rendering and multithreading approaches
- Applications, ongoing research projects
- Perspective and Conclusion



HAPTIC RENDERING AND MULTITHREADING APPROACHES

HAPTIC FEEDBACK OF SIMULATED PROCEDURE

- Stability: Haptic device is a robotic arm !! The control must be stable
  - ➡ When coupling with SOFA, simulation must be robust to any gesture of the user !
  - ➡ Related to passivity: the simulation and the control should never add energy to the system...
  - ➡ Time step must be as small as possible (as delay creates energy): often use of 500Hz / 1kHz

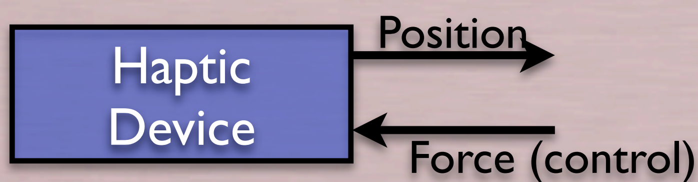
- Transparency: the force that is transmitted to the user is supposed to be the actual force, computed in the simulation
  - ➡ Computation of forces in the simulation must be correct + the control should not perturb the rendering
  - ➡ Problem with Damping forces; Damping helps the stability but is very bad for transparency
  - ➡ In our context, haptic rendering mainly comes from mechanical interactions: haptic algorithm must be as close as possible to interaction models

HAPTIC FEEDBACK OF SIMULATED PROCEDURE

- Use of «Impedance» (reversible) robots
- Direct coupling... no !
- Virtual coupling method
- Constraint-based approach (god-object approaches)

HAPTIC FEEDBACK OF SIMULATED PROCEDURE

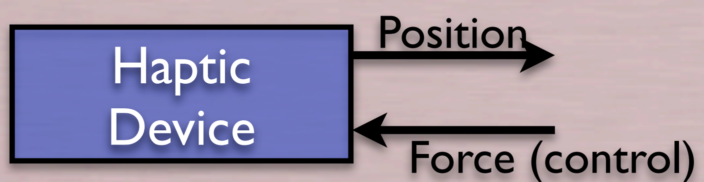
- Use of «Impedance» (reversible) robots



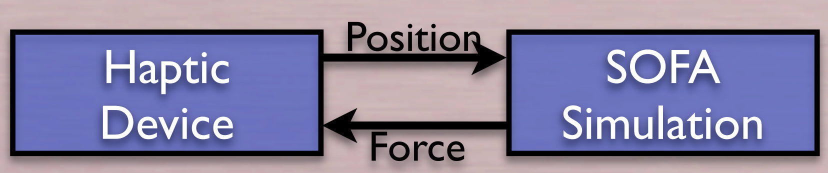
- Direct coupling... no !
- Virtual coupling method
- Constraint-based approach (god-object approaches)

HAPTIC FEEDBACK OF SIMULATED PROCEDURE

- Use of «Impedance» (reversible) robots



- Direct coupling... no !

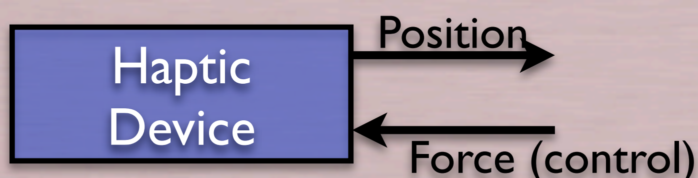


- Virtual coupling method

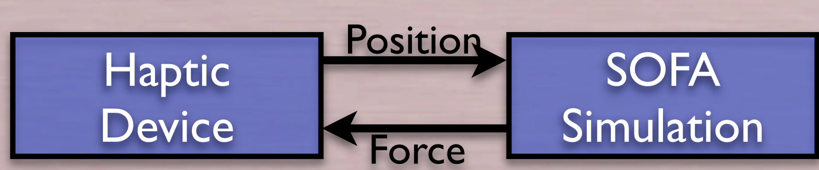
- Constraint-based approach (god-object approaches)

HAPTIC FEEDBACK OF SIMULATED PROCEDURE

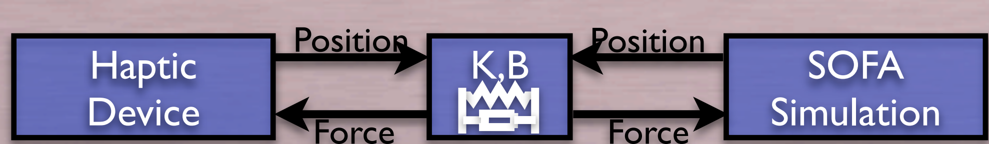
- Use of «Impedance» (reversible) robots

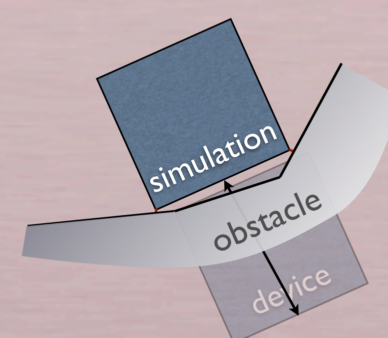


- Direct coupling... no !



- Virtual coupling method

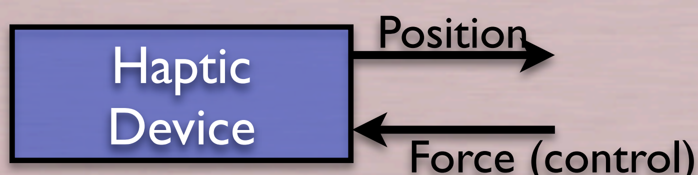




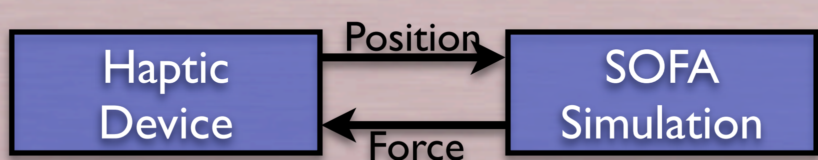
- Constraint-based approach (god-object approaches)

HAPTIC FEEDBACK OF SIMULATED PROCEDURE

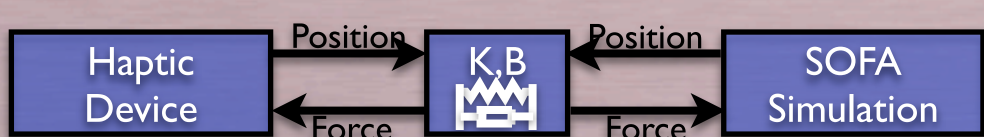
- Use of «Impedance» (reversible) robots

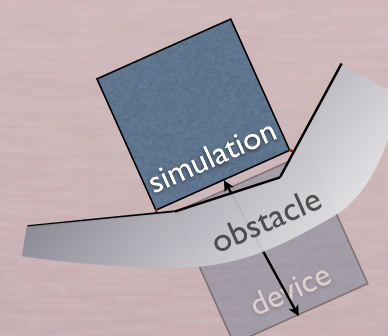


- Direct coupling... no !

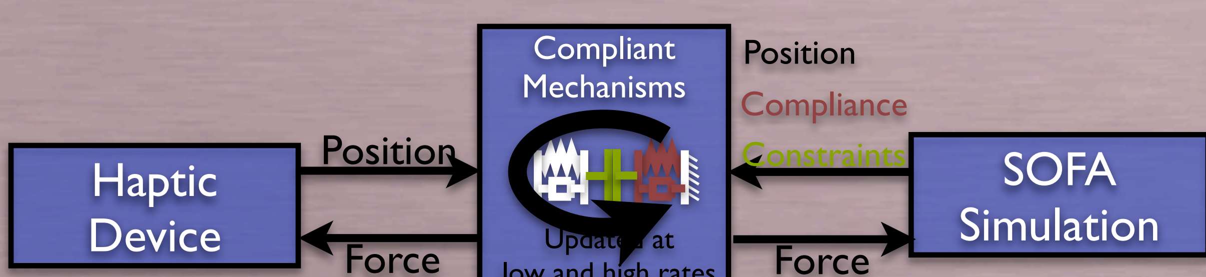


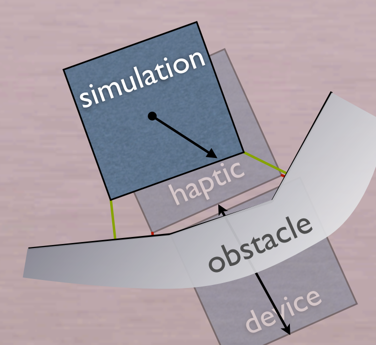
- Virtual coupling method





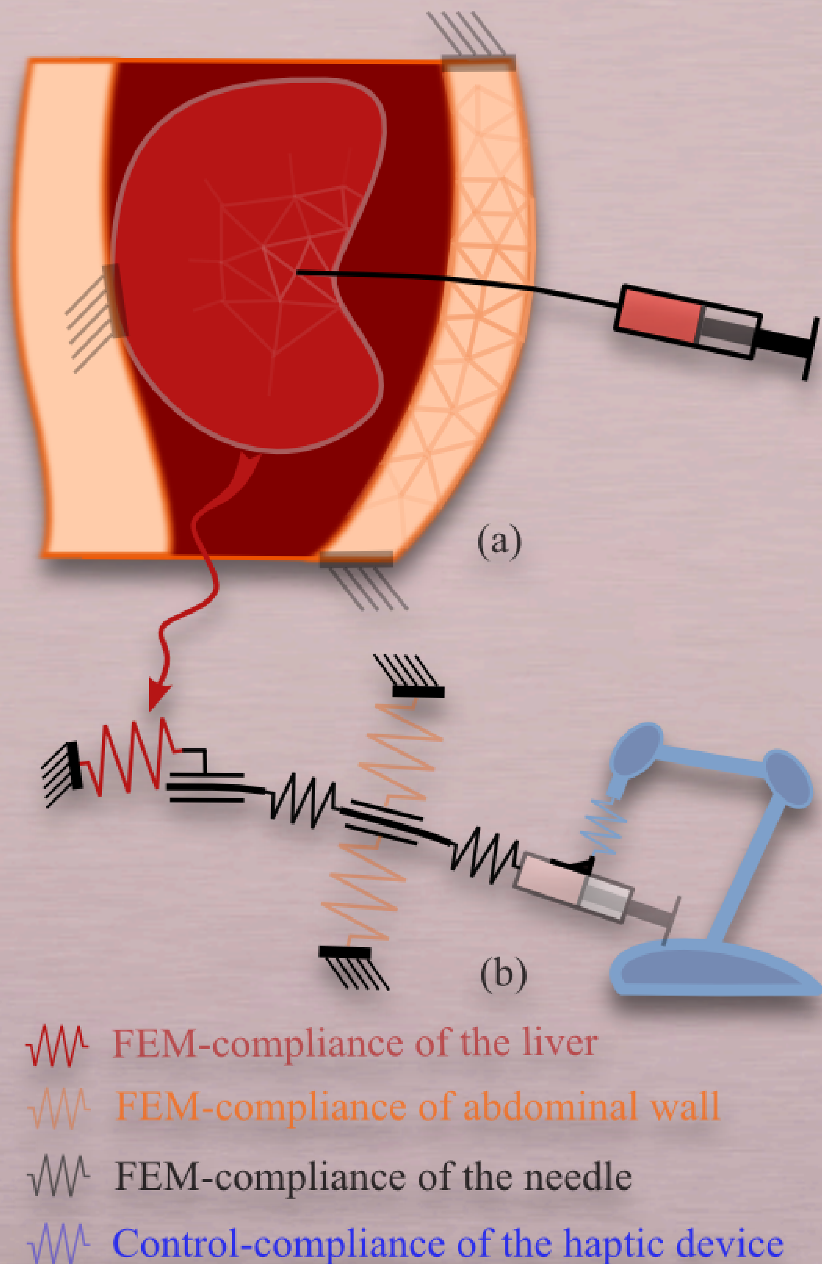
- Constraint-based approach (god-object approaches)





MULTIRATE COMPLIANT MECHANISMS

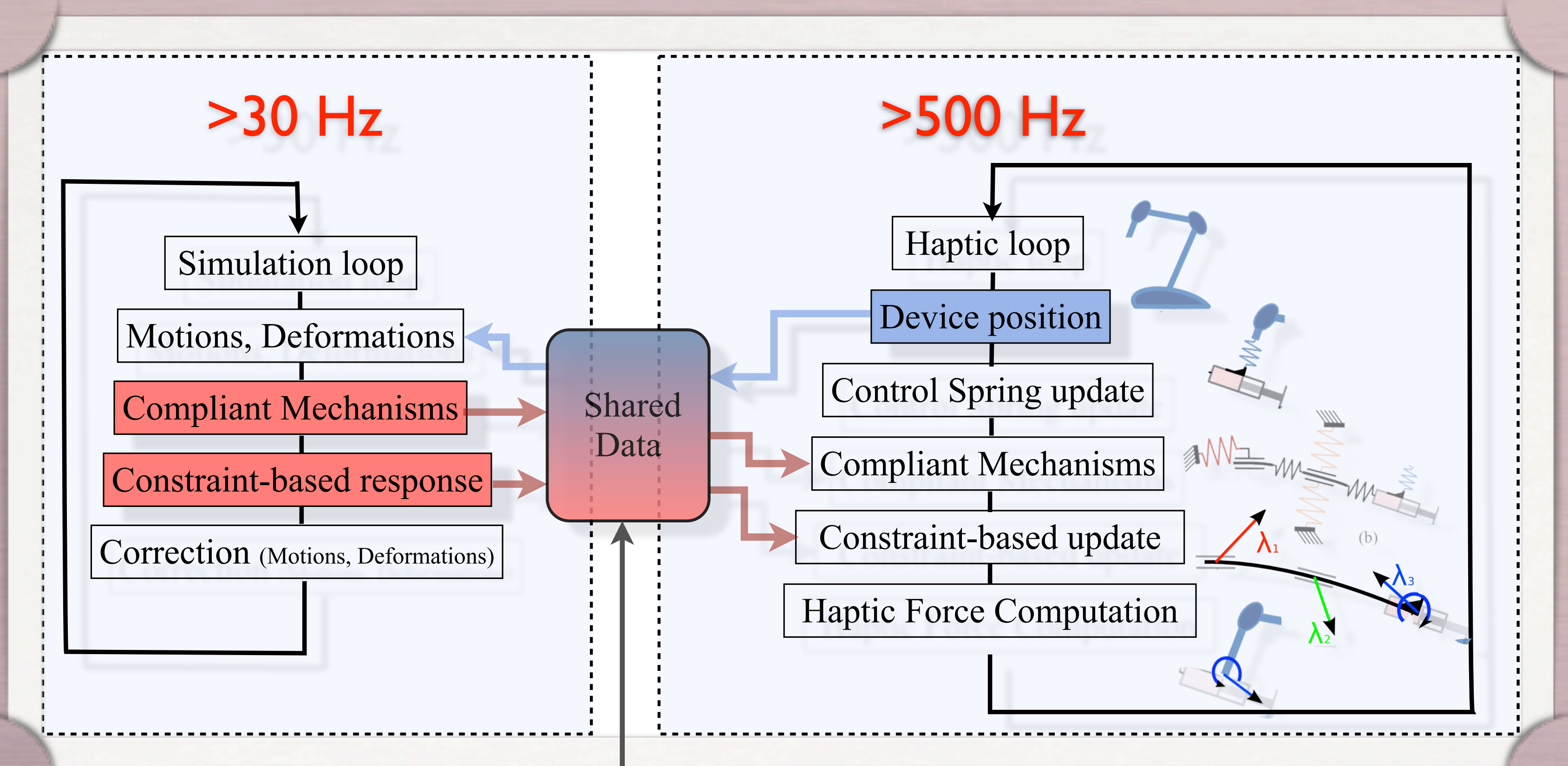
- Mechanisms
  - Support an extensive number of interaction types,
  - Versatile definition of constraint laws (adequate force/motion transmission model).
- Multirate
  - Build and simulate the mechanisms at low rates
  - Share with the haptic loop
  - Recompute at high rates for an intuitive and passive control.
- Compliant
  - Use the mechanical coupling between interaction spots,
  - Build compliance matrices based on physical models,
  - Handle both deformable and rigid objects.



Saupin et al. ISBMS 2008

Peterlik et al. IEEE Trans on Haptics 2011

MULTIRATE COMPLIANT MECHANISMS



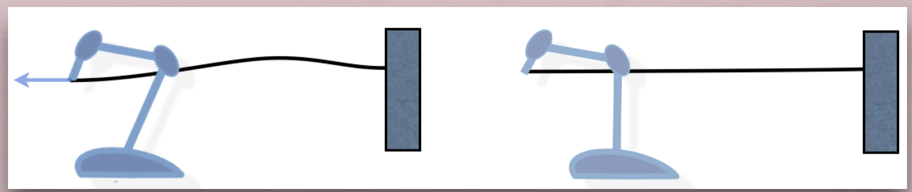
W, δ^{free}, λ and constraints law

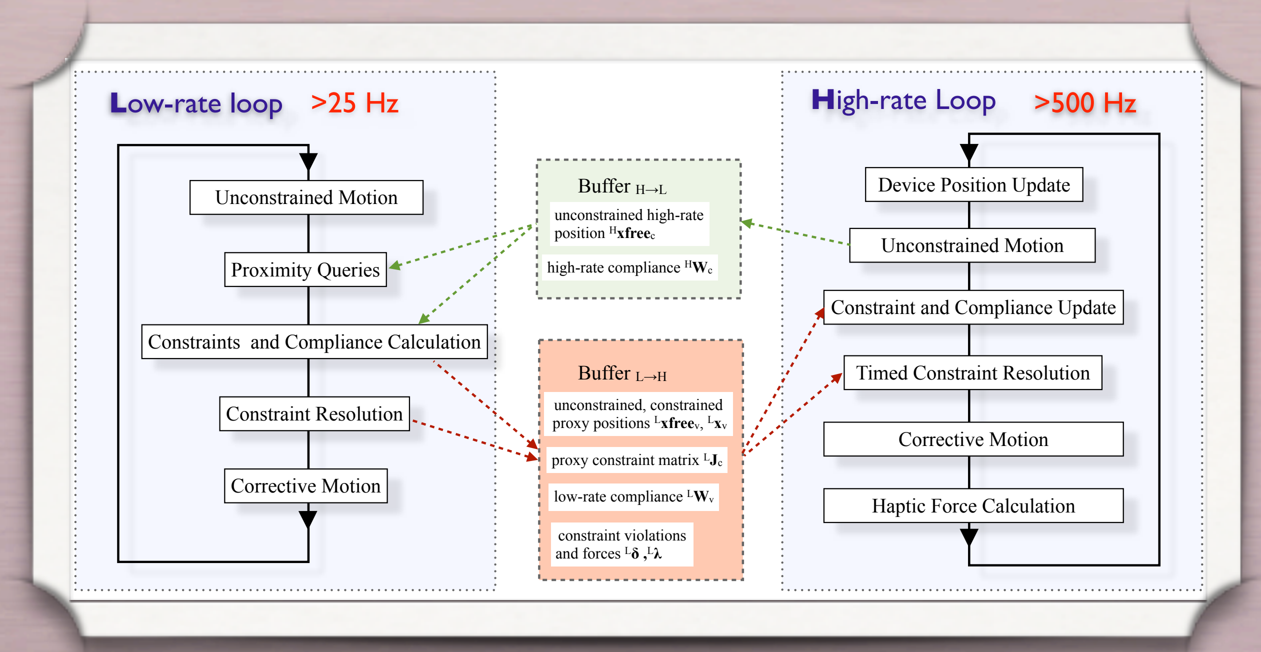
Saupin et al. ISBMS 2008

Peterlik et al. IEEE Trans on Haptics 2011

TOWARDS ASYNCHRONOUS SIMULATION

- Fast bending/streching transition
  - Wire needs to be simulated at high rates
  - The other deformable models are simulated at low rates
- Asynchronous strategy:
  - The thread is simulated at haptic rates (> 500 Hz)
  - The remaining of simulation at low rates (> 25 Hz)
- Preliminary results:
  - Constraints computed at both low and high rates !
  - about 20 beams at 1000Hz
  - Limited to quasi-static behaviors
  - Ongoing work on dynamic models

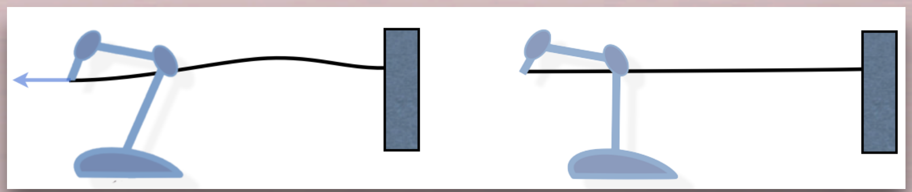


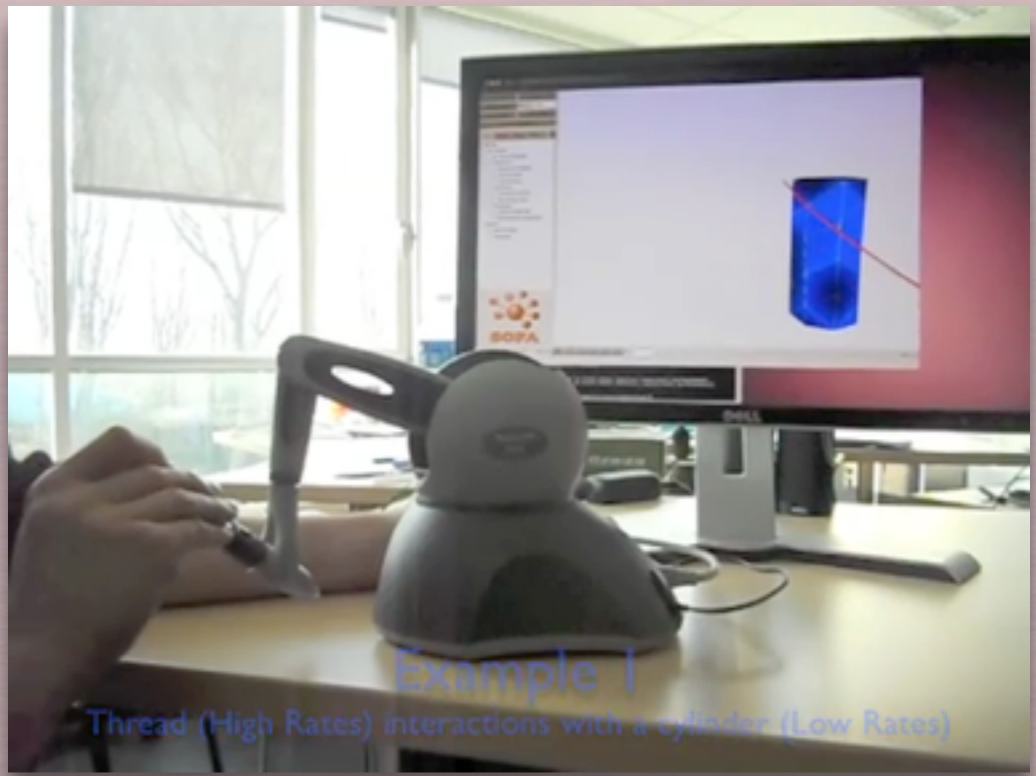


Peterlik et al. IROS 2011

TOWARDS ASYNCHRONOUS SIMULATION

- Fast bending/streching transition
  - Wire needs to be simulated at high rates
  - The other deformable models are simulated at low rates
- Asynchronous strategy:
  - The thread is simulated at haptic rates (> 500 Hz)
  - The remaining of simulation at low rates (> 25 Hz)
- Preliminary results:
  - Constraints computed at both low and high rates !
  - about 20 beams at 1000Hz
  - Limited to quasi-static behaviors
  - Ongoing work on dynamic models

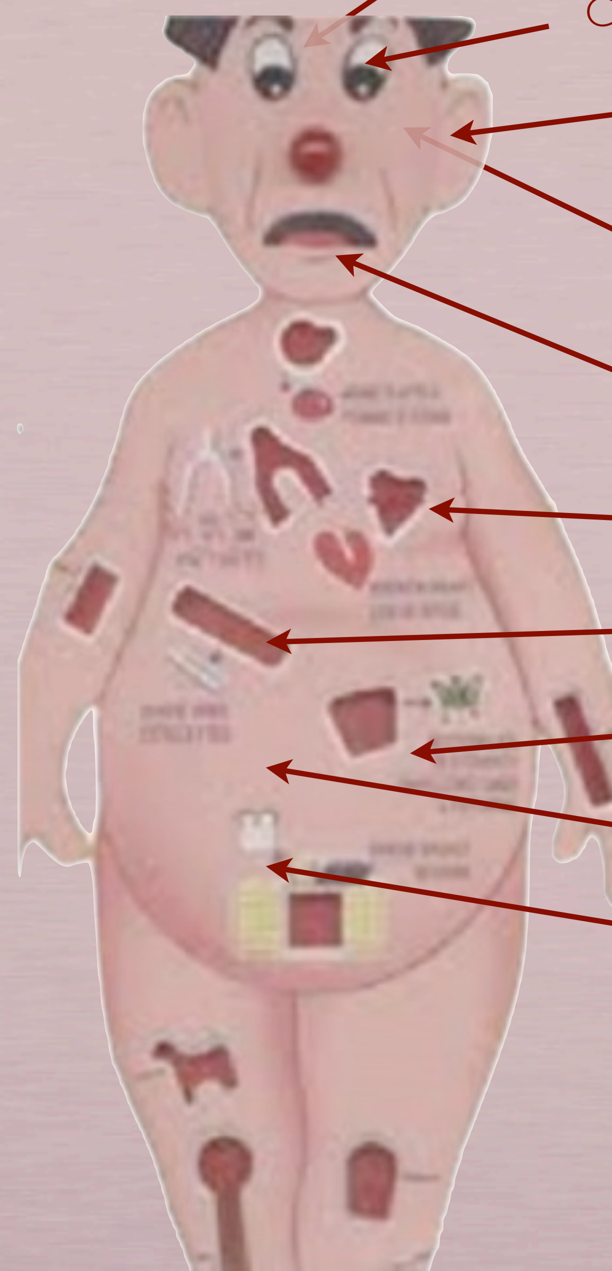




Peterlik et al. IROS 2011

- Haptic rendering and multithreading approaches
- Applications, ongoing research projects
- Perspective and Conclusion

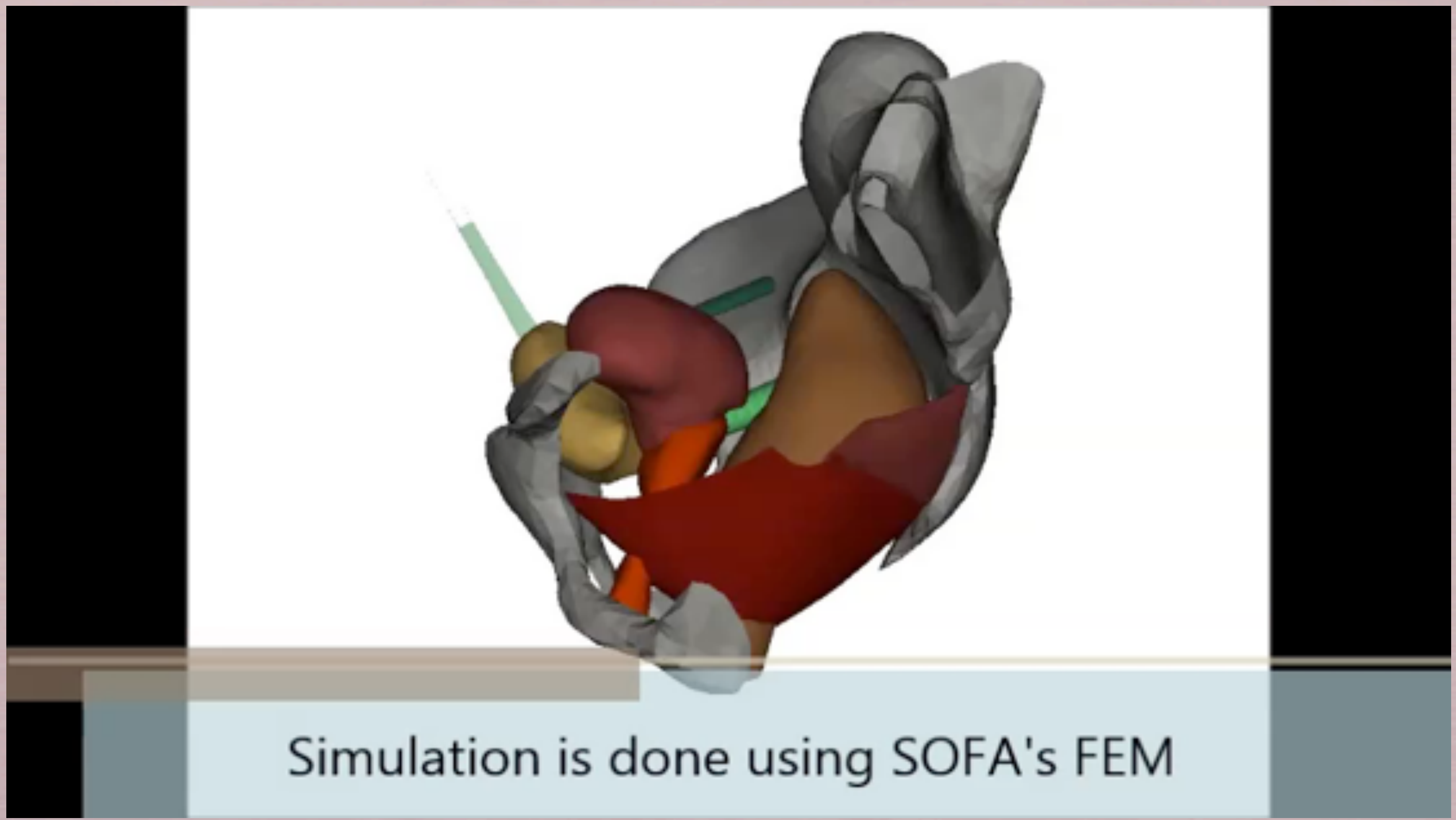
APPLICATIONS, ONGOING RESEARCH PROJECTS



APPLICATIONS, ONGOING RESEARCH PROJECTS

PLANNING SYSTEM FOR PROLAPSUS SURGERY

- Collaboration with CHRU Lille and LML
- Understand the prolapsus by simulation
  - LML experience on modeling: hyperelastic constitutive law based on hundreds of mechanical tests on tissues.
  - Simulation of the multi-organ pelvic system
  - Comparison between simulation and pathologic dynamic MRI
- Personalized simulation
  - Anatomical models from patient data
  - Inverse simulation for parameter identification
  - Influence on the surgery choices

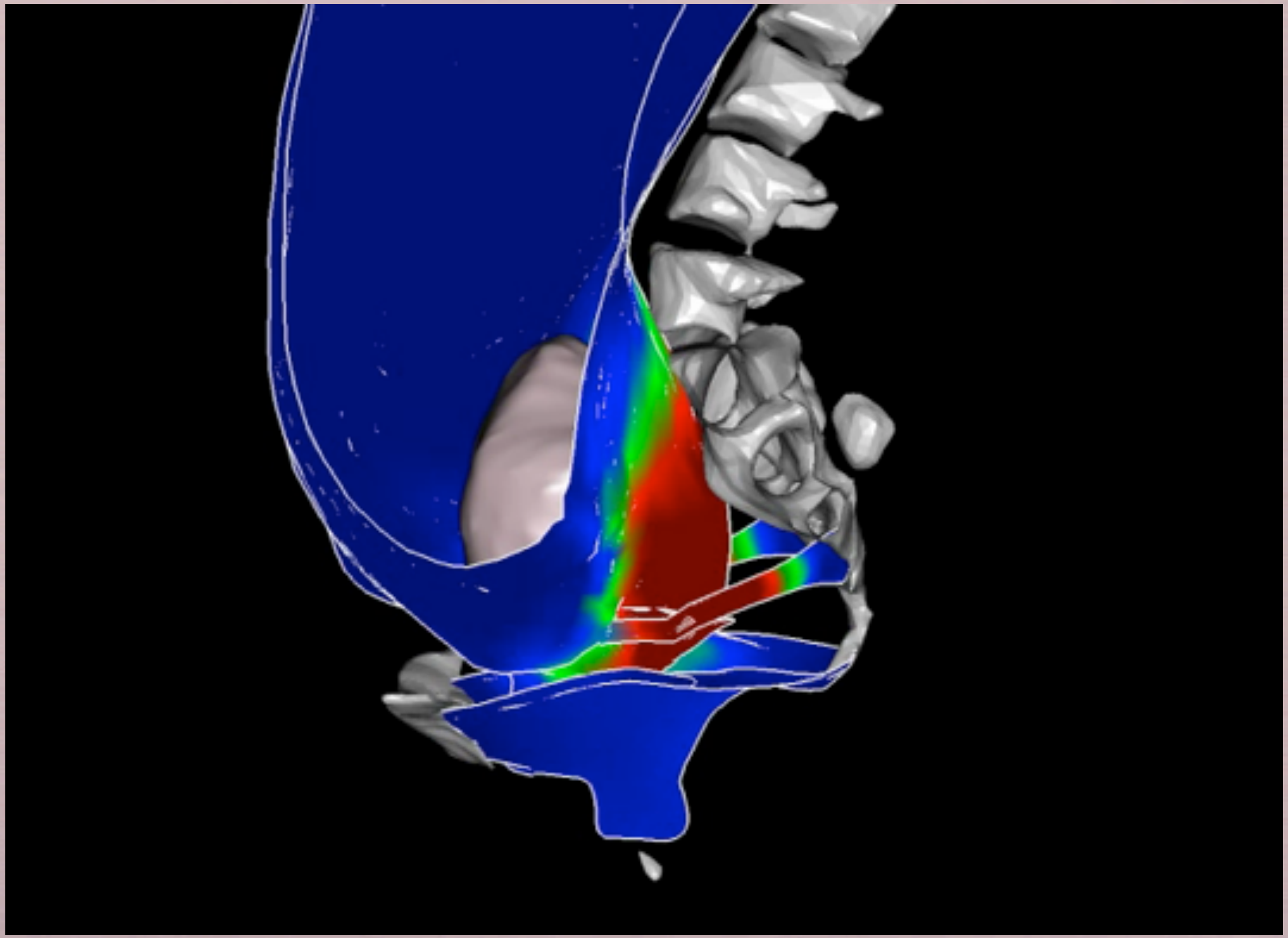


PLANNING SYSTEM FOR PROLAPSUS SURGERY

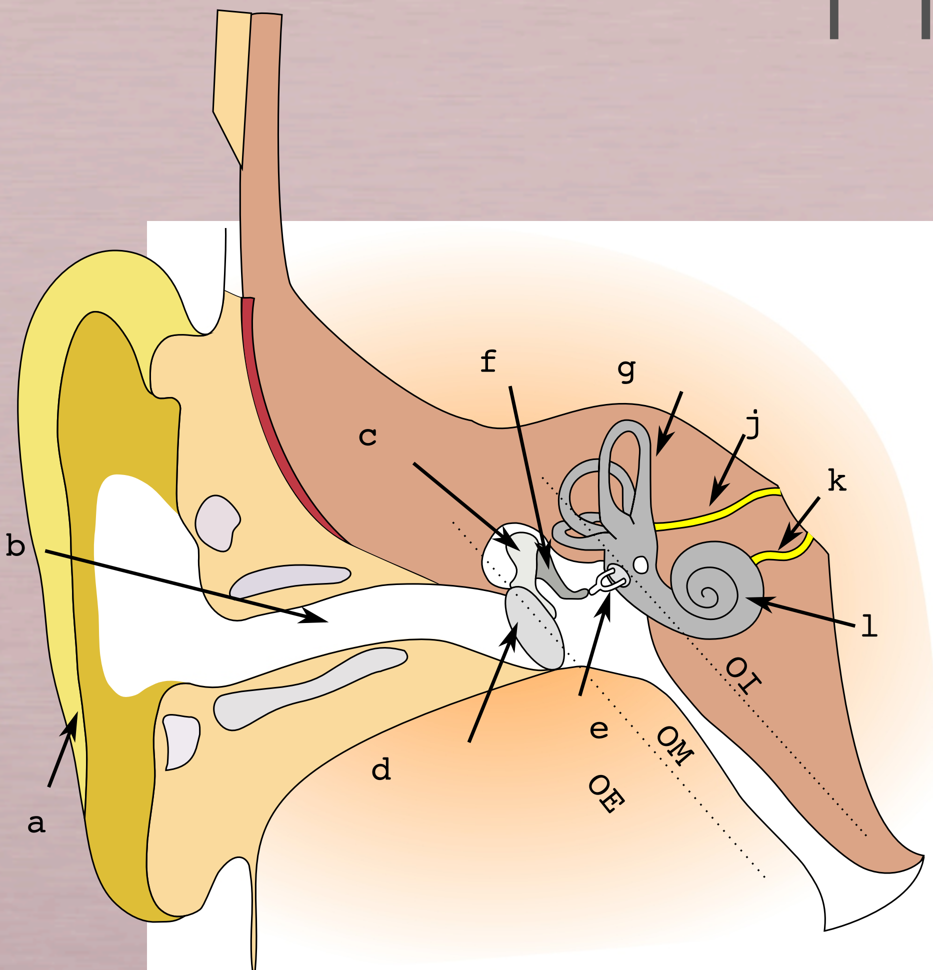
- Collaboration with CHRU Lille and LML
- Understand the prolapsus by simulation
  - LML experience on modeling: hyperelastic constitutive law based on hundreds of mechanical tests on tissues.
  - Simulation of the multi-organ pelvic system
  - Comparison between simulation and pathologic dynamic MRI
- Personalized simulation
  - Anatomical models from patient data
  - Inverse simulation for parameter identification
  - Influence on the surgery choices

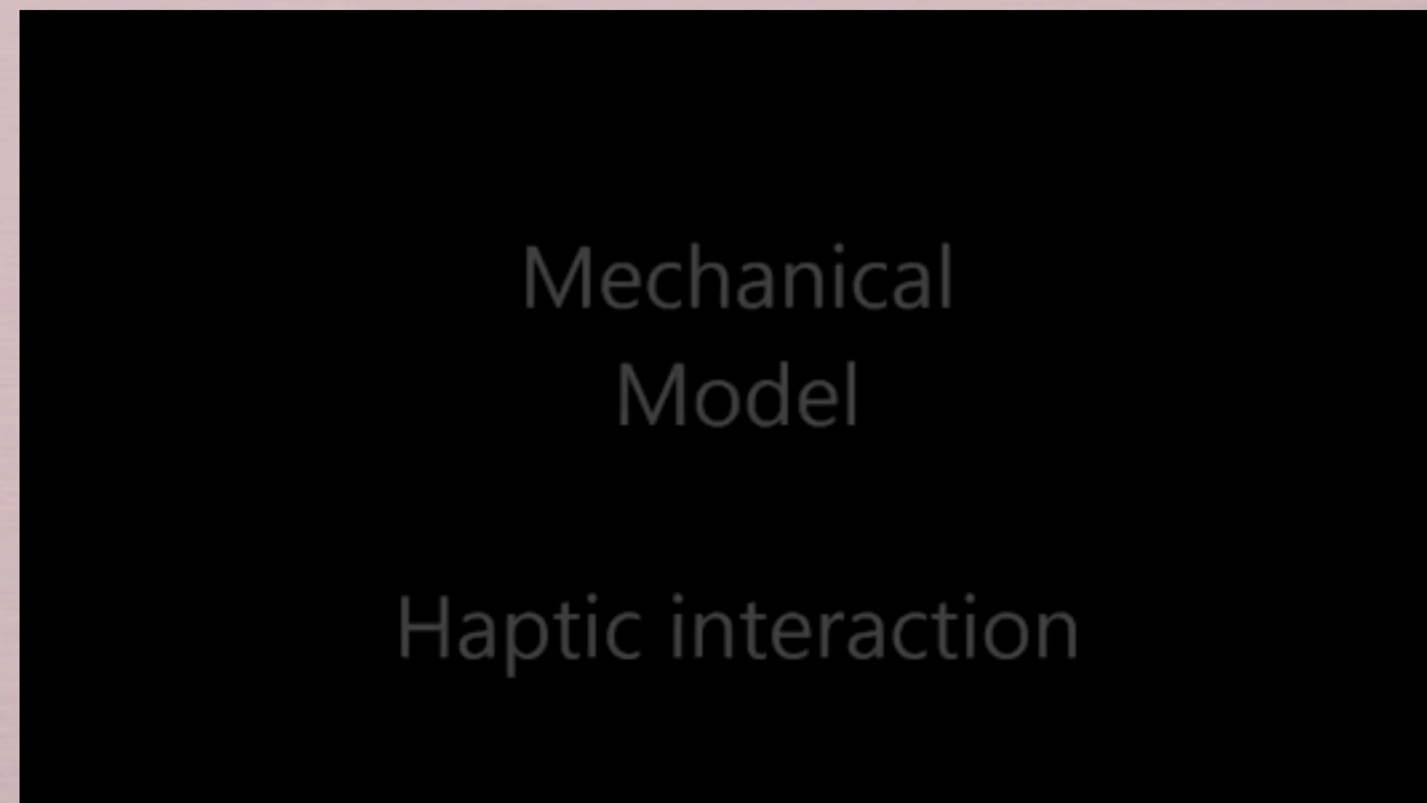
PLANNING SYSTEM FOR PROLAPSUS SURGERY

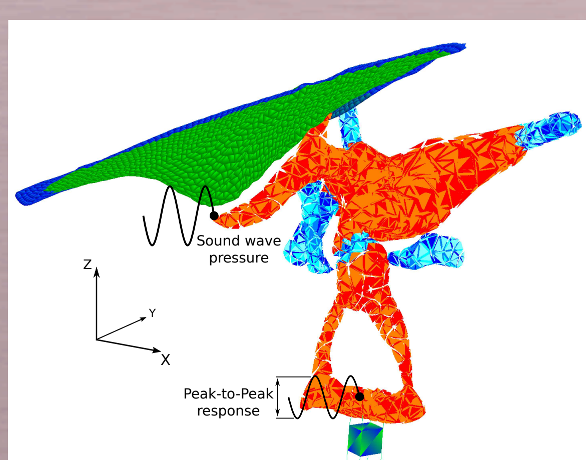
- Collaboration with CHRU Lille and LML
- Understand the prolapsus by simulation
  - LML experience on modeling: hyperelastic constitutive law based on hundreds of mechanical tests on tissues.
  - Simulation of the multi-organ pelvic system
  - Comparison between simulation and pathologic dynamic MRI
- Personalized simulation
  - Anatomical models from patient data
  - Inverse simulation for parameter identification
  - Influence on the surgery choices



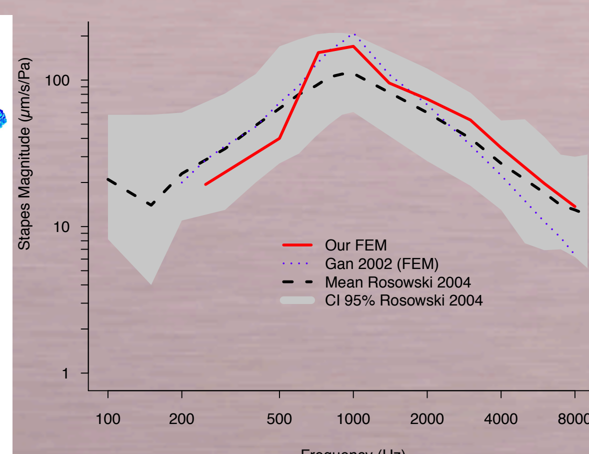
MIDDLE EAR SURGERY



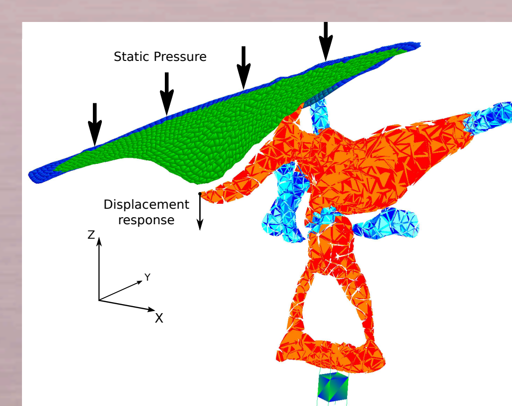




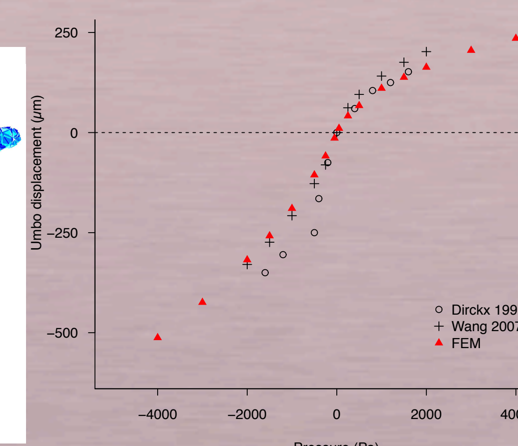
(a) Transfer function method.



(b) Transfer function results.

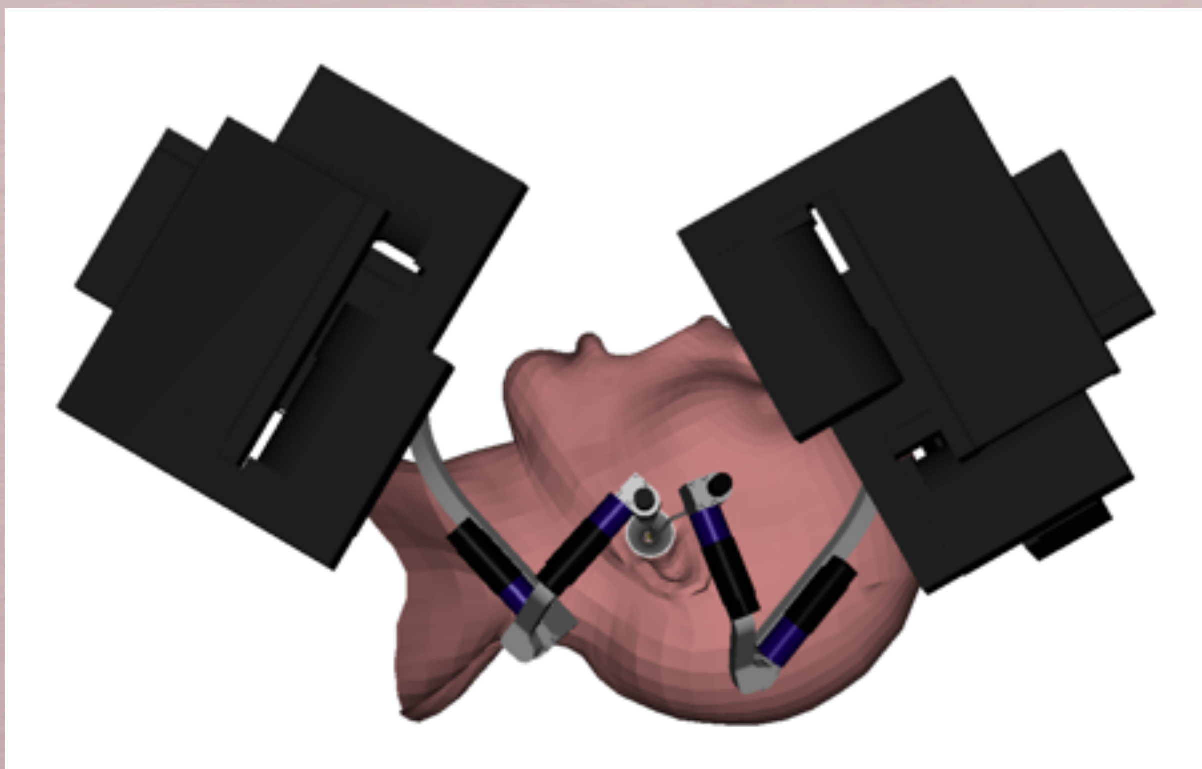


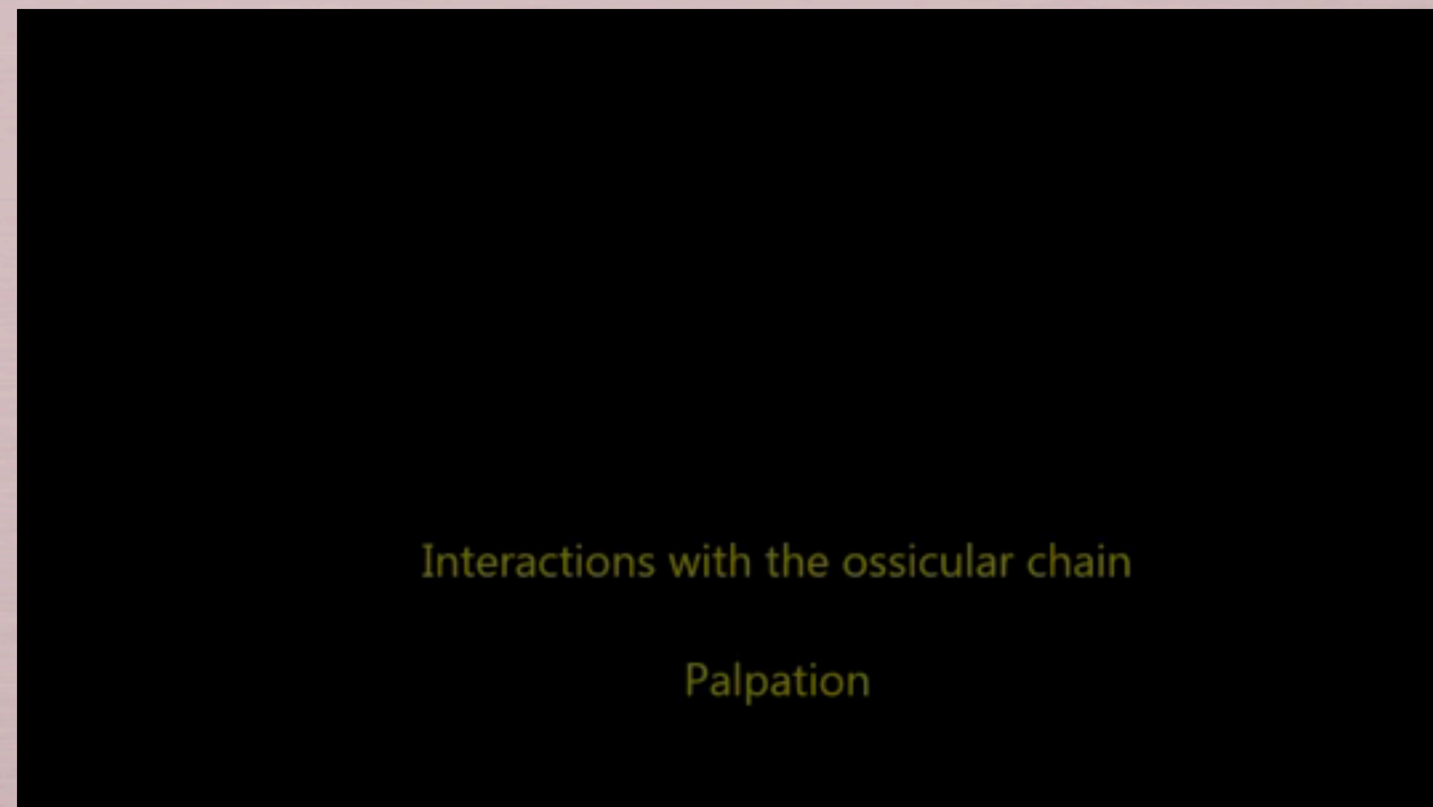
(a) Static pressure method.

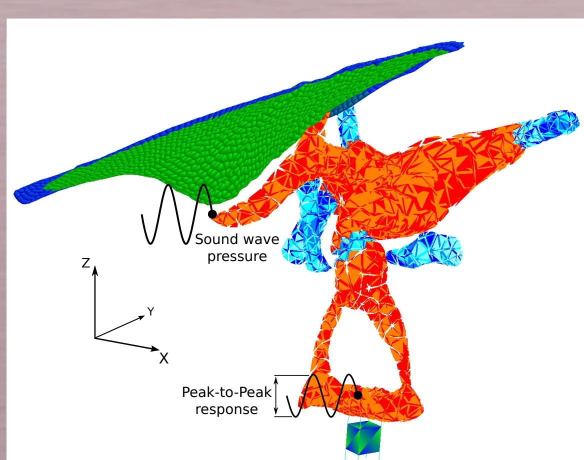


(b) Static pressure results.

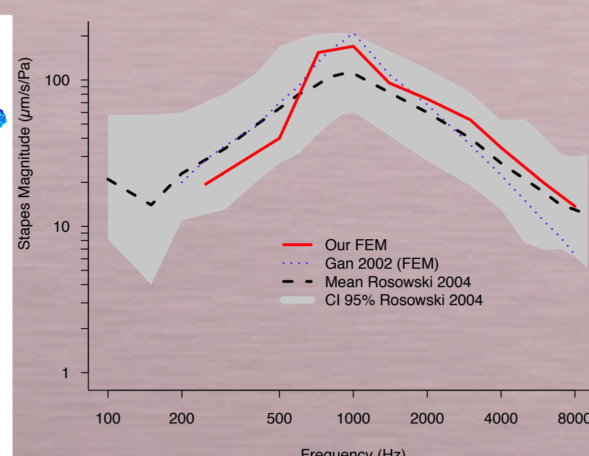
MIDDLE EAR SURGERY



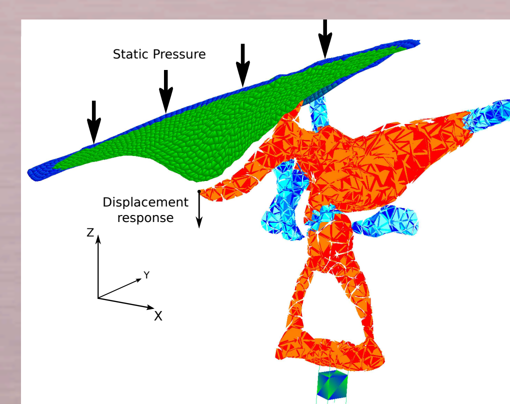




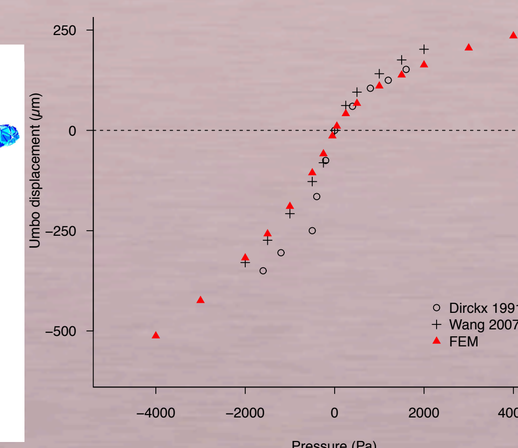
(a) Transfer function method.



(b) Transfer function results.



(a) Static pressure method.



(b) Static pressure results.

- Applications, ongoing research projects
- Perspective and Conclusion

PERSPECTIVE AND CONCLUSION

PERSPECTIVE AND CONCLUSION

NEW NUMERICAL CHALLENGES...

- Towards real-time predictive simulation
  - Include more sophisticated models from mechanics with measurable parameters
  - Better management of the computing ressources
  - Online estimation of the numerical errors.
  - Improve and assess the code quality
- From patient data to simulation models...
  - Adapt segmentation / registration tools to simulation requirements
  - Use new modalities of images (Ultrasound / MRI Elastography) to fuse geometrical and available mechanical information.

NEW NUMERICAL CHALLENGES...

- Adapt the numerical methods to the needs of the simulation
  - Better distribution of the computational effort
  - Adaptive discretization (solve the problem of cutting...)
  - Dynamic adaptations of the models
- Accuracy and validation
  - Validate the implementation
  - Online estimation of the numerical errors
  - Influence of «small» structures

PARAMETRIZATION AND (REAL-TIME) INVERSE METHODS

- Patient bio-mechanical parameters
  - Models should have less parameters / parameters that we could really obtain !
  - Extend «elastography» to interacting non-linear models
  - For per-operative guidance: on-line correction of the parameters & simulation course
    - no divergence with reality
    - the simulation remains predictive for the following step of the procedure

- Bridge with robotics...
  - Plan the robotic procedures using simulation
  - Deformable robots in a surgical environments
    - Difficulties of control
      - Many degrees of freedom
      - Interaction with deformable anatomy
    - Preliminary results using simulations

(patent deposit in progress...)

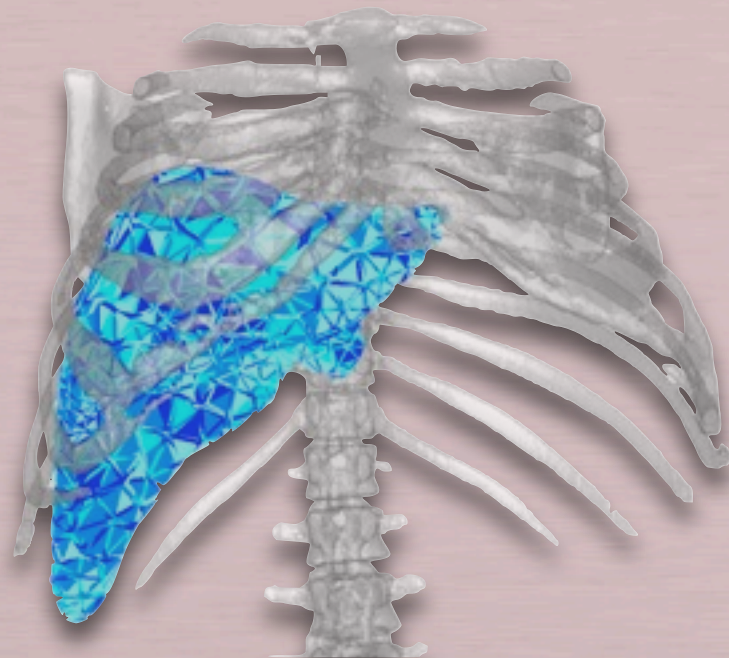
PARAMETRIZATION AND (REAL-TIME) INVERSE METHODS

- Patient bio-mechanical parameters
  - Models should have less parameters / parameters that we could really obtain !
  - Extend «elastography» to interacting non-linear models
  - For per-operative guidance: on-line correction of the parameters & simulation course
    - no divergence with reality
    - the simulation remains predictive for the following step of the procedure

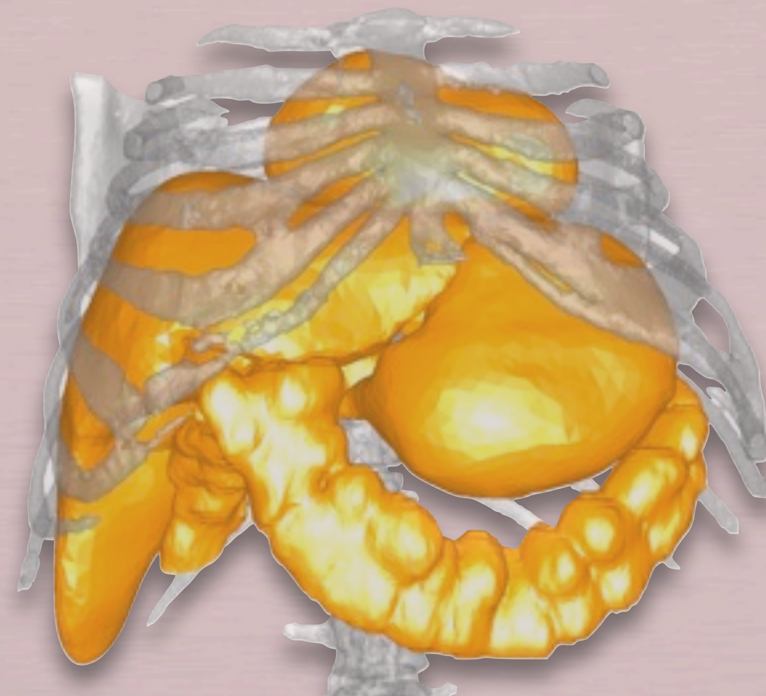
- Bridge with robotics...
  - Plan the robotic procedures using simulation
  - Deformable robots in a surgical environments
    - Difficulties of control
      - Many degrees of freedom
      - Interaction with deformable anatomy
    - Preliminary results using simulations

(patent deposit in progress...)

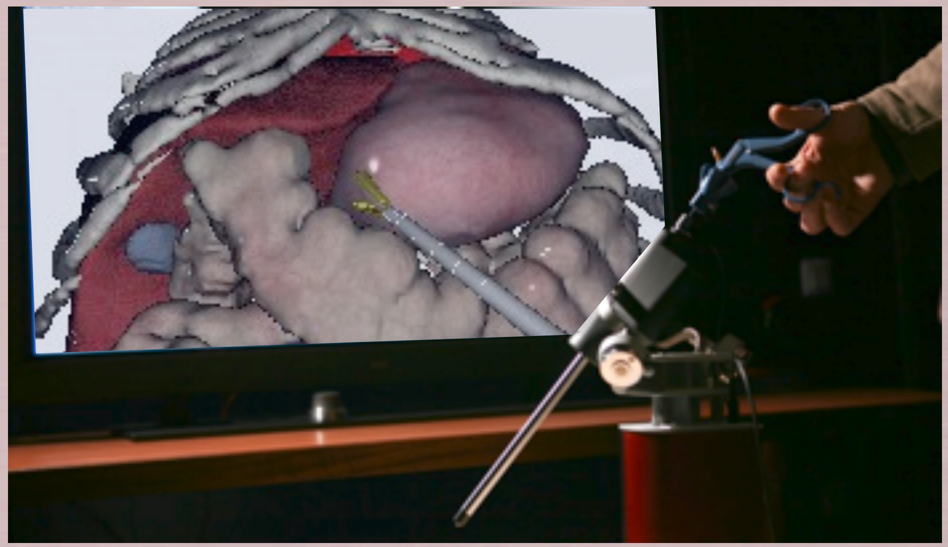
ANY QUESTION ?



deformable models



Mechanical interactions



Haptics

applications

Conclusion